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An analytical study of laminar co-current flow gas absorption through a parallel-plate gas–liquid membrane contactor

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Abstract

A mathematical formulation for a laminar co-current gas absorption process was developed in a parallel-plate gas–liquid membrane contactor that independently regulates gas and liquid flows. Physical carbon dioxide absorption in water was studied. The analytical solutions for such a conjugated Graetz problem were solved for fully developed laminar velocity profiles using the orthogonal expansion technique in power series. This work investigates the influence of the absorbent flow rate, gas feed flow rate and the CO_2 concentration in the gas feed on absorption efficiency.

The theoretical predictions show that the initial CO_2 concentration effect in the gas feed on the absorption efficiency is more significant than that from the gas feed flow rate variation as the absorbent flow rate is specified. A higher absorbent flow rate has a positive effect producing better absorption efficiency in the case when the CO_2 concentration in gas feed is specified.

Keywords: Gas absorption; Membrane contactor; Mass transfer; Parallel-plate system; Graetz problem

1. Introduction

Gas absorption is a mass transfer operation in which soluble gas mixture components are dissolved in a liquid when the two phases are in contact with each other. In conventional gas absorption processes, the gas mixtures are usually dispersed and in direct contact with liquids in absorption devices such as packed or plate columns. Dispersion in these contact devices has many shortcomings like foaming, unloading and flooding. A substitutive process that can overcome these disadvantages is non-dispersive contact via a microporous membrane.

In most physical absorption processes the overall mass transfer coefficient depends on the liquid film resistance. Membranes, usually microporous, hydrophobic and non-selective, are employed as a contact interface with the two phases making contact in the pore mouth. The separation efficiency of this kind of procedure depends on the distribution coefficient of gas solute in the two phases. Because the absorbent liquid flows on one side and the gas flows on the other side of the membrane, this method is useful in applications where the required solvent/feed ratio is very high or very low. Therefore, membrane gas absorption processes have gained increasing attention in recent years [6]. Qi and Cussler [7,8] investigated the absorption of a number of gases including carbon dioxide using various solvents in a hydrophobic microporous hollow fiber membrane contactor. Karoor and Sirkar [5] used the hollow fiber membrane contactor to separate carbon dioxide and nitrogen with either pure water or an aqueous amine solution. Some investigators [1–3,10] considered different absorbents or different hollow fiber modules to absorb carbon dioxide from gas mixtures. Mavroudi et al. [4] studied the absorption of carbon dioxide using water or diethanolamine in a microporous hollow fiber membrane contactor with non-wetted and partial wetted mode. Dindore et al. [9] investigated the gas absorption using a hollow fiber membrane contactor at elevated pressure.

Like laminar heat and mass transfer at steady state with negligible axial conduction or diffusion for multi-stream problems, the membrane gas absorption process is also known as conjugated Graetz problem. Nunge and Gill [12,13] used the orthogonal expansion technique to solve some heat or mass transfer problems in counterflow systems. Ho et al. [14,15] studied the heat and mass transfer through a parallel-plate channel

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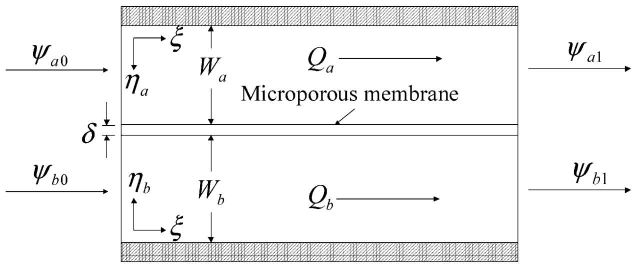

Fig. 1. Parallel-plate membrane gas–liquid contactor.

with recycle by using an orthogonal expansion technique. Many experimental data or numerical solutions for membrane gas absorption processes were discussed in past literature. However, the number of analytical solutions for this process is few.

In this study, we wish to use the orthogonal expansion technique to find the analytical solutions for the membrane gas absorption problem. The gas feed is a mixture of carbon dioxide and nitrogen, and the liquid absorbent is pure water. These solutions enable us to better understand the importance of the membrane gas–liquid contactor at various parameters.

2. Theory

2.1. Mathematical statements

Consider a parallel-plate co-current flow membrane gas–liquid contactor with length L, width B, distance between membrane and plate W_a and W_b, respectively, as shown in Fig. 1. The thickness of the hydrophobic microporous membrane is δ. Gas feed and absorbent flow occur in different channels. The liquid phase pressure is slightly higher than that of the gas phase and less than the membrane breakthrough pressure. The overall mass transfer process includes three steps. First, the solute gas transfers into the membrane surface from the bulk gas phase. It then transfers through the membrane pores. Finally, the solute gas transfers into the bulk liquid from the membrane–liquid interface. The following assumptions were utilized for the theoretical formulation: (1) steady state fully developed flow in each channel; (2) negligible axial diffusion and conduction; (3) isothermal operation and constant physical properties; (4) the applicability of Henry's law; (5) the membrane is completely gas-filled; (6) negligible the evaporated water from liquid stream into the membrane pores.

Using these assumptions, the velocity distributions and conservation equations in dimensionless form may be described as

$$\left[\frac{v_a W_a^2}{L D_{\mathrm{AB}}}\right]\frac{\partial\psi_a(\eta_a,\xi)}{\partial\xi}=\frac{\partial^2\psi_a(\eta_a,\xi)}{\partial\eta_a^2},$$

$$v_a(\eta_a)=\bar{v}_a(6\eta_a-6\eta_a^2) \tag{1}$$

$$\left[\frac{v_b W_b^2}{L D_{\mathrm{AC}}}\right]\frac{\partial\psi_b(\eta_b,\xi)}{\partial\xi}=\frac{\partial^2\psi_b(\eta_b,\xi)}{\partial\eta_b^2},$$

$$v_b(\eta_b)=\bar{v}_b(6\eta_b-6\eta_b^2) \tag{2}$$

in which

$$\bar{v}_a=\frac{Q_a}{W_a B},\quad \bar{v}_b=\frac{Q_b}{W_b B},\quad \eta_a=\frac{x_a}{W_a},\quad \eta_b=\frac{x_b}{W_b},$$

$$\xi=\frac{z}{L},\quad \psi_a=\frac{C_a}{C_{a0}-C_{b0}},\quad \psi_b=\frac{C_b}{C_{a0}-C_{b0}},$$

$$Gz_a=\frac{Q_a W_a}{4LBD_{\mathrm{AB}}},\quad Gz_b=\frac{Q_b W_b}{4LBD_{\mathrm{AC}}}. \tag{3}$$

The boundary conditions for solving Eqs. (1) and (2) are

$$\psi_a(\eta_a,0)=\psi_{a0} \tag{4}$$

$$\psi_b(\eta_b,0)=\psi_{b0} \tag{5}$$

$$\frac{\partial\psi_a(0,\xi)}{\partial\eta_a}=0 \tag{6}$$

$$\frac{\partial\psi_b(0,\xi)}{\partial\eta_b}=0 \tag{7}$$

$$\frac{\partial\psi_a(1,\xi)}{\partial\eta_a}=-\frac{\varepsilon W_a}{\delta}\left[\psi_a(1,\xi)-\frac{1}{H}\psi_b(1,\xi)\right], \tag{8}$$

$$\frac{\partial\psi_a(1,\xi)}{\partial\eta_a}=-\frac{D_{\mathrm{AC}}W_a}{D_{\mathrm{AB}}W_b}\frac{\partial\psi_b(1,\xi)}{\partial\eta_b} \tag{9}$$

where ε is the permeability of membrane and H is dimensionless Henry's law constant [11].

The above equations show that the boundary conditions for both channels are not independent. This type of problem is called the conjugated Graetz problem. The analytical solutions may be solved using of an orthogonal expansion technique.

The variables are separated in the form

$$\psi_a(\eta_a,\xi)=\sum_{m=0}^{\infty}S_{a,m}F_{a,m}(\eta_a)G_m(\xi) \tag{10}$$

$$\psi_b(\eta_b,\xi)=\sum_{m=0}^{\infty}S_{b,m}F_{b,m}(\eta_b)G_m(\xi) \tag{11}$$

where $S_{i,m}$ is the expansion coefficient associated with the eigenvalue λ_m and eigenfunction $F_{i,m}$ and G_m is a function will be damped out exponentially with ξ. Applying Eqs. (1) and (2) lead to

$$G_m(\xi)=\mathrm{e}^{\lambda_m\xi} \tag{12}$$

$$F''_{a,m}(\eta_a)-\frac{v_a W_a^2\lambda_m}{LD_{\mathrm{AB}}}F_{a,m}(\eta_a)=0 \tag{13}$$

$$F''_{b,m}(\eta_b)-\frac{v_b W_b^2\lambda_m}{LD_{\mathrm{AC}}}F_{b,m}(\eta_b)=0 \tag{14}$$

and the boundary conditions in Eqs. (6)–(9) can be written as

$$F'_{a,m}(0)=0 \tag{15}$$

$$F'_{b,m}(0)=0 \tag{16}$$

$$S_{a,m}F'_{a,m}(1)=-\frac{\varepsilon W_a}{\delta}\left[S_{a,m}F_{a,m}(1)-\frac{1}{H}S_{b,m}F_{b,m}(1)\right] \tag{17}$$