Separation and Purification Technology 165 (2016) 114–122

Contents lists available at ScienceDirect

Separation and Purification Technology

journal homepage: www.elsevier.com/locate/seppur

The permeability prediction of beds of poly-disperse spheres with applicability to the cake filtration

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ARTICLE INFO

Article history:
Received 23 December 2015
Received in revised form 9 March 2016
Accepted 28 March 2016
Available online 29 March 2016

Keywords:
Cake filtration
Poly-disperse particles distribution
Effective/average particle diameter
Cake permeability prediction (experimental/models)

ABSTRACT

The interactions between the cake and depth filtration of poly-disperse spherical particles is examined by comparing experimentally measured cake permeability to the permeability predictions of analytical models. In the experiments, the influence of the cake forming history on the cake permeability is investigated, where the parameters as (i) different filtration materials, (ii) test suspension flow rates, and (iii) particle concentrations in the suspension are varied. The permeability models are given as product of preset constant, porosity function and square of characteristic particle size. For the poly-disperse porous media, the characteristic particle size has to account for the distribution of particle sizes which is typically accomplished through the use of various moments of the distribution. Clearly, the size distribution function of particles forming the cake has to be utilized which is obtained after correcting the original distribution function of particles used in the test suspension for the particles which pass through the cake. This implies that the particles have to be counted after the test suspension passes the filtering material. Following this framework, a set of experiments is carried out to determine the permeability of poly-disperse cake. For each experiment, the permeability is also evaluated analytically using four different long-established models in combination with different averages for the particle diameter of the poly-disperse particle sample, trying to identify an averaging rule for which the analytical predictions are most close to the experimental results.

1. Introduction

Filtration is widely used to separate particles from gases or liquids in numerous industries including petroleum, food, pharmaceutical and automotive, to name just a few. In the majority of these applications, in addition to particle entrapment within the porous filter medium itself, a layer of particles, called a cake, is formed at the upstream surface of the filtering medium and contributes substantially to the flow resistance of the filter. Prediction of cake permeability is clearly an important aspect of filtration process design. There are two major difficulties with predicting cake permeability. First, in the majority of applications, the particles comprising the cake are poly-disperse in size requiring determination of an average, effective, particle size if a mono-disperse model is to be employed. Second, in many applications, the cake is formed in the second stage of the filtration process, with the first being depth filtration. Finally, some particles pass all the way through the filter and cake, so that the cake permeability is due to a particle size distribution different from that of the upstream suspension.

For slow flows, the fluid velocity is proportional to the pressure gradient resulting from viscous interaction with the surfaces of the medium. That is, the flow obeys Darcy's law. The proportionality constant is the ratio of the permeability to the fluid viscosity [1]. In the groundwater sciences the expression hydraulic conductivity [2,3] and in the cake filtration community the expression (specific) cake resistance [4,5] are commonly used to denote, respectively, the permeability or its inverse. Many different expressions for the permeability exist in the literature. Common to all is that the permeability is expressed as a product of a preset constant, specific geometrical function and particle size. The best known example is perhaps the Carman-Kozeny relation [6]. The geometrical function in the permeability models is calculated from either equivalent medium consisting of parallel tubes further corrected for non-circular cross section and the fluid flow tortuous paths [1,4,7–10], or by representing the porous medium as a periodic arrays of mono-disperse spheres [11,12]. The preset constant in the permeability models varies stipulating that there are additional

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http://dx.doi.org/10.1016/j.seppur.2016.03.046
1383-5866/

morphological parameters that should be taken into account in predicting the permeability. A broad variability of the Kozeny constant is summarized in Tien and Ramarao [13], where it is argued that same porosity functionality is used for porous media which are morphologically very different, i.e. spherical, cylindrical, fibers or consolidated media of different types.

Although porous media are mainly heterogeneous, an attempt can be made to generalize models developed for mono-disperse media to poly-disperse ones, by exploiting averaged, effective size scales (diameters), D_m, and D_{mn}, based on the particle size distribution function moments, M_m, and ratios of moments, M_{mn} (see next section for notations and explanations of the moments). Poly-disperse models using the three means, harmonic [14], geometric [15,16], and arithmetic [17], are also reported in the literature. Models using many varieties of moment ratios are available. Notably in the filtration literature, all the ranges of average particle diameters from D_{21} up to D_{43} have been used, i.e., D_{21} in [18], D_{31} in [19], D_{43} in [20], and the Sauter diameter, D_{32} in [4,5,14,21–23]. Endo and Alonso [19] formulated a model for a log-normal distribution also including a shape factor for non-spherical particles and a so-called void function. Their result is similar to using D_{31} with log-normally distributed sizes in the Kozeny-Carman formula. Additionally, in some studies the cut off diameter of 10% smallest particles [3], and the median particle size diameter [15] are used to calculate the poly-disperse medium permeability. The different permeability models and particle size averages which give the best agreement to the experimental data are used in these comparisons. It can be seen that there is a large variability in average particle size used, implying that in poly-disperse media, besides the permeability model constant, the particle size distribution affects the permeability value.

In this study, poly-disperse filter cake permeability is measured experimentally and compared to the predictions of mono-disperse permeability models. In the models, the particle diameter is replaced by an average, effective particle diameter found from the particle size distribution function. For a broad particle size distribution, calculated average particle diameters can vary as much as two orders of magnitude, which produce even higher differences in calculated permeability as it is a quadratic function of the selected particle size average. Due to the fact that some of the smallest particles pass through the filter, the particle size distribution function of suspended particles differs to some extent from the distribution function in the filtration cake; the same applies for the averaged diameters. Hence, the goal of this investigation is twofold: (i) to perform a detailed experimental study of the influence of the history of the cake formation on its permeability (e.g., cake formed on top of various porous filter media, cake formed from higher and lower suspension concentration, etc.), and (ii) to use a detailed set of experimental data for poly-disperse media to determine a proper averaging rule (a moment) which, for the available permeability models, provides the best fit to the experimental results.

2. Permeability model

The slow flow through a porous medium is described by Darcy's law, where the permeability quantifies how easily fluid can flow through a porous medium. The permeability is an intrinsic material property of the porous medium depending only on the medium geometry [23] including porosity, tortuosity and particles sizes and their distribution functions [24–26], and not on the nature of the fluid. In predicting the permeability, probably the most used empirical model is the Kozeny-Carman formula. It is based on a combination of Hagen-Poiseuille and Darcy law for steady, laminar and incompressible flow through a bundle of circular capillary tubes [17]. The famous formulation is given as [6]:

$$K = \frac{\phi^3}{k_K(1-\phi)^2} d_p^2 \tag{1}$$

in which ϕ is the porosity, d_p is the particle diameter and k_K is the Kozeny constant, an empirical constant which is often reported having different values greatly underlining the uniqueness of Eq. (1) and suggesting that the medium morphology should be taken into account in the permeability predictions [13]. This becomes clearer once looking onto an alternate form of Eq. (1) which is given by Panda and Lake [17]:

$$K = \frac{\phi^3}{2\tau(1-\phi)^2 S_V^2} \tag{2}$$

in which τ is the tortuosity, and S_V is the specific surface area equal to the ratio of the particle surface to the particle volume, i.e. equal to $S_V = 6/d_p$ for a spherical particle of diameter d_p. Eq. (2) has been modified in many different ways, one being by setting different values for tortuosity [17], or by introducing additional parameters as threshold porosity to account for close packing when void spaces start losing connectivity. Another permeability model for spherical particles has been formulated by Happel [27], where:

$$K = \frac{d_p^2}{18\varphi} \frac{3 - 4.5\varphi^{1/3} + 4.5\varphi^{5/3} - 3\varphi^2}{3 + 2\varphi^{5/3}} \tag{3}$$

with $\varphi = 1-\phi$ being the fraction of solid phase.

In Eq. (2), the specific area, S_V, is defined for the mono-disperse spherical particles which is directly related to the particle diameter. Similarly, the specific area can be defined for the poly-disperse sample, where from the particle number distribution function, the distribution moment of order m and m is an integer, M_m, is calculated from known weights, w_i, and particle diameters, $d_{p,i}$. The weight of each particle size is calculated from the number of particles, n_i, and total number of particles in the sample, N:

$$M_m = \sum_i w_i d_{p,i}^m, \quad \text{where } w_i = \frac{n_i}{N} \tag{4}$$

(i.e. for $m = 2$, M_2 is arithmetic average of particles surface area). The ratio of two moments is defined as $M_{mn} = M_m/M_n$, and from the moments, the average particle diameter is calculated from $D_m = {M_m}^{1/m}$ and $D_{mn} = {M_{mn}}^{1/(m-n)}$. The specific area (expressed per unit volume) can be calculated from the distribution moments as follows:

$$S_V = \frac{A}{V} = 6\frac{\sum_i w_i d_{p,i}^2}{\sum_i w_i d_{p,i}^3} = 6\frac{1}{D_{32}} \tag{5}$$

So S_V is proportional to the reciprocal of the Sauter diameter and can be used directly in the Kozeny equation, Eq. (2). This generalization shows at least two limitations, the first being the value of the tortuosity, which in the poly-disperse sample, may be changed by small particles nesting between larger ones. The second limitation is caused due to the flow dissipation in heterogeneous medium producing an effective medium which is different from one that consists of particles of equal sizes. Thus, other average particle diameters can fit better in the Kozeny-Carman formula for the permeability of poly-disperse media. A similar reasoning can be applied to the Happel permeability model, as well as two additional empirical models for poly-disperse media, proposed by Rumpf and Gupte [28] and Garcia et al. [29] using the Sauter and harmonic mean diameters respectively:

$$K_{RG} = \frac{\phi^{5.5}}{5.6} D_{32}^2 \quad \text{and} \quad K_G = 0.11\phi^{5.6} d_h^2. \tag{6}$$