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An analytical approach to predict pressure drop and collection efficiency of dust-load pleated filters

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ABSTRACT

In this work, a series of first-principle analytical expressions are derived to predict the instantaneous pressure drop and collection efficiency of pleated filters under dust loading condition. Both the depth and surface filtration regimes are formulated for filters with triangular and rectangular pleats. The analytical expressions derived in this paper can be used in the early stages of designing a pleated filter to circumvent the need for conducting CPU-intensive numerical calculations. The predictions of our analytical expressions are compared with those reported in previous studies and good agreement is observed.

1. Introduction

Pressure drop and collection efficiency of a pleated filter depend on a number of parameters including, but not limited to, pleat geometry, flow properties, fiber diameter(s), filter porosity, and particle inertia. In addition, the particle deposition pattern in the pleat channel or inside the fibrous structure of the filtration media can greatly affect the filter's overall pressure drop and collection efficiency. Accurate prediction of the instantaneous performance (i.e., pressure and collection efficiency) of a pleated filter in service requires CPU-intensive parallel computation. In a previous study, we developed a microscale simulation method (i.e., simulations on scales comparable to the fiber diameter) that could predict the pressure drop and collection efficiency of a dust-loaded fibrous sheet without requiring any empirical correction factors [1,2]. This CPU-intensive simulation method was then modified to make it applicable to simulation domains as large as a pleat channel, and was referred to as the macroscale simulation method [3–5]. The macroscale simulations are significantly faster than their microscale counterpart; however, they need to be calibrated using experiment or computational data generated by a more accurate simulation method (e.g., microscale simulations). To further reduce the computational cost of designing a pleated filter, we recently developed a semi-numerical method that could be considered "CPU-independent" in terms of calculation time [6,7]. This was thanks to the empirical correlations that were used to describe the velocity field in the simulation domain, leaving only the particle trajectory calculations to be carried numerically by the semi-numerical model.

Using the information obtained from the above-mentioned simulations with regards to dust-cake growth patterns inside a pleated filter, the current paper presents a novel set of analytical expressions that can be used to predict the instantaneous performance of a pleated filter during its service life without the need to use a computer. This is achieved by determining the independent variables that affect the performance of a pleated filter, and by grouping them together in such a way that they can be related to the filter's pressure drop and collection efficiency (dependent variables). Our work also takes advantage of the findings of many pioneering studies reported in the literature (see e.g., [8–17]).

In the following sections, we first focus on dust-loaded pleated filters with triangular pleats in the depth filtration regime—the regime in which particles deposit inside the fibrous media—and derive predictive expressions for their pressure drop and collection efficiency (Section 2). We then extend our formulations to also develop such expressions for pleated filters operating in the surface filtration regime, i.e., filters which do not accommodate depth deposition (Section 3). These formulations are then modified for when the pleated geometry is formed into a circular shape (Section 4). Studies similar to those presented in Sections 2 and 3 are presented in Sections 5 and 6, respectively, for filters with rectangular pleats. Comparisons with experiment and our previous macro-scale models are presented in Section 7. The conclusions drawn from our work are given in Section 8.

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Nomenclature

Variables

c^c	Cunningham slip correction factor
d_f	fiber diameter (m)
d_p	particle diameter (m)
D	particle diffusivity ($\mathrm{m}^2\,\mathrm{s}^{-1}$)
E_0	filtration efficiency of a clean filter
E_D	single fiber efficiency due to Brownian diffusion
E_I	single fiber efficiency due to impaction
E_m	filtration efficiency of dust-loaded pleated filter
E_R	single fiber efficiency due to interception
E_Σ	total single fiber efficiency
f	ratio of the cake thickness at the pleat entrance to that near the pleat end
h	height of pleat channel (m)
k_m	permeability constant of fibrous media (m^2)
k_c	permeability constant of granular media (m^2)
K^0	modified zeroth order Bessel function of the second kind
K^1	modified first order Bessel function of the second kind
Ku	Kuwabara factor
l	length of pleat channel (m)
m_p	mass of deposited particles (kg)
m_p^{max}	maximum value for mass of particles deposited in fibrous media (kg)
n_p	total number of particles deposited inside a pleated media
p	pressure (Pa)
Pe	Peclet number
r_i	pleat inlet radius (m)
r_o	pleat outlet radius (m)
R	interception parameter (particle to fiber diameter ratio)
t_m	thickness of fibrous media
T	temperature (K)
St	Stokes number
U	inlet air velocity upstream of filter (m/s)
V_c	volume of dust cake (m^3)
V_m	volume of fibrous media (m^3)
V_p	volume of a particle (m^3)
v_w	filtration velocity inside media (m/s)
v_w^f	filtration velocity near pleat end (m/s)
v_w^i	filtration velocity at pleat entrance (m/s)
w	wall length in triangular pleats (m)
Δp	pressure drop (Pa)
Δp_0	pressure drop of clean pleated filter (Pa)
Δp_d	pressure drop due to particle deposition inside fibrous media (depth filtration) (Pa)
Δp_l	pressure drop across dust loaded pleated filter (Pa)
Δp_s	pressure drop due to particle deposition upstream the fibrous media (surface filtration) (Pa)
Δp_v	pressure drop due to viscous effects in pleat channel (Pa)
β	resistance to flow in porous media
β_c	resistance to flow in dust-cake
β_l	resistance to flow in fibrous media loaded with particle deposits
β_m	resistance to flow in clean fibrous media
φ	solid volume fraction
φ_v	solid volume fraction of dust-cake
φ_m	solid volume fraction of clean fibrous media
φ_p	solid volume fraction of particles inside a fibrous media
φ_p^{max}	maximum volume fraction of particles deposited in a fibrous media
δ	cake thickness (m)
δ_e	cake thickness measured at pleat end (m)
δ_f	cake thickness when cake reaches pleat's centerline (m)
δ_i	cake thickness measured at pleat entrance (m)
γ	pleat half-angle (°)
ρ_p	particle density ($\mathrm{kg\,m}^{-3}$)
ρ_w	water density ($\mathrm{kg\,m}^{-3}$)
σ	Boltzmann constant ($\mathrm{kg}^{-2}\,\mathrm{m}^2\,\mathrm{K}^{-1}$)
μ	air viscosity ($\mathrm{kg\,m}^{-1}\,\mathrm{s}^{-1}$)

Superscript or subscripts

c	Cunningham
f	location where cake reaches pleat centerline
i	pleat entrance
0	clean filter
c	cake
d	depth filtration
e	pleat end
l	fibrous media loaded with particles
m	fibrous media
o	pleat outlet
p	particle
s	surface filtration
w	water

2. Triangular pleats in depth filtration regime

Assuming filter media to be the sole source of pressure drop in a pleated filter, the pressure drop of a clean filter can be obtained using Darcy's law (validity of this assumption is examined later in this paper):

$$\Delta p_0 = v_w \frac{\mu}{k_m} t_m = v_w \beta_0 \tag{1}$$

where Δp_0 and v_w are the initial pressure drop and the filtration velocity (velocity through the fibrous media) across the fibrous media, respectively. k_m and t_m are the permeability constant and thickness of the media, respectively, and μ is the air viscosity. The parameter $\beta_0 = \Delta p_0 / v_w$ is pressure drop per unit velocity or "resistance" of the clean fibrous media.

In order to obtain the ratio of the filtration velocity to the inlet velocity (velocity at the pleat entrance), we consider the mass conservation law for a control volume over the pleat channel as shown in Fig. 1a. Therefore, we obtain

$$w v_w = hU \tag{2}$$

where w is the filtration area per unit depth (normal to the page), U is the inlet velocity, and h is the pleat height. When particles deposit inside a fibrous media, they cause additional resistance to the flow; the pressure drop of a particle-loaded media can be obtained as

$$\Delta p_l = v_w \beta_l \tag{3}$$

where β_l is the resistance of the dust-loaded media. Using the un-weighted resistivity method [18,19], an expression can be developed for the total resistance of a dust-loaded media to air flow, i.e.,

$$\beta_l = \beta_m + \beta_p = \frac{\mu t_m}{k_m(\varphi_m)} + \frac{\mu t_m}{k_c(\varphi_p)} \tag{4}$$