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A model for transient cross flow filtration in a narrow rectangular domain

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ABSTRACT

Analytical solutions for flow rate, pressure, and permeate flux, which vary with time and distance, are developed for a cross flow filter in a narrow rectangular domain with either one or two permeable walls. The governing equations consist of a generalized Darcy's law combined with flow and continuity equations. The contribution to fouling as either a static surface cake or depth plugging in the media is governed by a distribution function representing the ratio of the two phenomena. The mathematical model and solution for the clean filter case (i.e. no fouling) are presented first, along with design parameters (characteristic length, flow rate, and pressure) that arise from the solutions. The analytical solutions for the no-fouling case yield limiting values of filter length, inlet flow rate, and inlet pressure drop. These bounds also provide reasonable limits for operating parameters in the transient model with fouling. Parameter studies that compare transient effects with fouling to the clean filter case are presented to illustrate the application of the model.

1. Introduction

In this paper, a model for cross flow filtration in a rectangular domain with either 1 or 2 permeable walls is developed. Forms of the analytical solution for pressure in the clean-filter case similar to the solution presented in this work can also be found in [1] (for a domain with two permeable walls) and [2] (for domains with either one or two permeable walls). Experimental and modeling results for a rectangular cross flow filter with one permeable wall are compared in [3,4]. The latter two works each assume a constant pressure drop throughout the filter length. Surface cake deposition in a rectangular domain is the focus of [5,6], while others, including [7,8,9] have studied internal fouling of the media. The findings in [9] indicate that in some scenarios the primary fouling phenomena is cake deposition, while in other cases in-depth pore plugging plays a dominant role. A recent paper, [10], provides an in-depth review of models incorporating blocking filtration laws for membrane filtration. The author urges the scientific community to further develop filtration models which are simple yet effective.

In the present work, governing equations and analytical solutions are developed for a narrow rectangular (slit) domain with transient fouling effects. Of particular interest are distance and time-dependent solutions for the flow rate, pressure, and permeate flux. The analytical solutions for the clean filter are expressed in terms of design parameters representing a reference length, flow rate, and pressure drop. The clean filter solutions are also used to derive a maximum filter length as well as intervals specifying allowable values for inflow rate and pressure drop. The limiting values for the no-fouling case provide a safe and reasonable operating envelope for the transient case with fouling.

The rest of this paper is organized as follows. In the next section, the governing equations and solutions for the clean rectangular cross flow filter, analogous to the model for the cylindrical filter in [11], are presented. Then this model is extended to the case in which fouling occurs due to formation of a stable static cake on the filter surface, and/or internal media fouling. Fundamental to this model are a time-dependent distribution function representing the ratio of cake to depth filtration, [12], and a generalization of Darcy's law to incorporate effects of cake deposition and depth plugging, [13]. Two sets of parameter studies that compare transient effects with fouling to the clean filter case are presented to illustrate the application of the model.

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Nomenclature

A	filter surface area (m^2)
$C(x,t)$	slurry concentration at position x and time t (kg/m^3)
C_0	slurry concentration at filter inlet (kg/m^3)
$C_d(t)$	concentration of dispersed phase material trapped in filter media (kg/m^3)
d	filter flow domain half-height (m)
H	filter media thickness (m)
K_c	composite parameter related to cake deposition (m/s)
K_d	composite parameter related to depth plugging ($kg/(m^3$-s))
L	filter length (m)
L_∞	maximum filter length, transient case (m)
$L_{\infty,\text{clean filter}}$	maximum filter length, no fouling case (m)
n_w	number of permeable walls (1 or 2)
$P(x,t)$	pressure inside filter at position x and time t (Pa)
P_0	pressure at filter inlet (Pa)
P_∞	permeate discharge pressure (Pa)
$Q(x,t)$	slurry volumetric flow rate inside filter at position x and time t (m^3/s)
Q_0	volumetric flow rate at filter inlet (m^3/s)
Q_{ref}	reference flow rate (m^3/s)
t	time (s)
W	filter width (m)
x	distance from filter inlet (m)
α	empirical model parameter related to cake bed thickness (dimensionless)
$\beta(t)$	filter resistance factor (dimensionless)
Γ	specific media permeability (m^2)
ΔP_{ref}	reference pressure drop (Pa)
ε	porosity of filter media (dimensionless)
η_g	gravimetric separation efficiency (kg/kg) (dimensionless)
$\Theta(t)$	gravimetric ratio $\frac{\text{mass in cake}}{\text{mass in pores}}$ (dimensionless)
λ	characteristic length (m)
μ	slurry viscosity (Pa-s)
$\xi(t)$	cake thickness (m)
ρ_c	cake density (kg/m^3)
ρ_d	density of dispersed phase (kg/m^3)
σ	empirical model parameter related to depth plugging (dimensionless)
Ψ	empirical model parameter related to permeate contamination (dimensionless)
$\varphi(x,t)$	permeate flux at position x and time t ($m^3/(s\text{-}m^2)$)

2. Mathematical model

2.1. Mathematical model for a clean filter

The modeling region is shown in Fig. 1. It is assumed that the width and half-height of the flow domain W and d, respectively, and the thickness of the filter media, H, satisfy the relationship in equation (1):

$$H, d \ll W \tag{1}$$

so that transverse flow and pressure changes are negligible in comparison to those in the main flow (i.e. x) direction.

Let n_w represent the number of permeable walls, i.e. $n_w = 1$ if there is 1 permeable wall (as shown in Fig. 1) or $n_w = 2$ if there are two permeable walls, on opposite sides of the domain. It is assumed that the permeation velocity is small in comparison to the channel (axial) velocity and does not significantly alter the axial flow profile. Assuming laminar incompressible flow with a no-slip condition along the channel boundaries and constant fluid properties, the flow rate $Q(x)$, pressure $P(x)$, and permeate flux $\varphi(x)$ satisfy these equations:

$$\frac{dQ}{dx} + n_w W \varphi(x) = 0 \tag{2}$$

$$P(x) - P_\infty = \frac{\mu H}{\Gamma}\varphi(x) \tag{3}$$

$$Q(x) + \frac{2}{3}\frac{d^3 W}{\mu}\frac{dP}{dx} = 0 \tag{4}$$

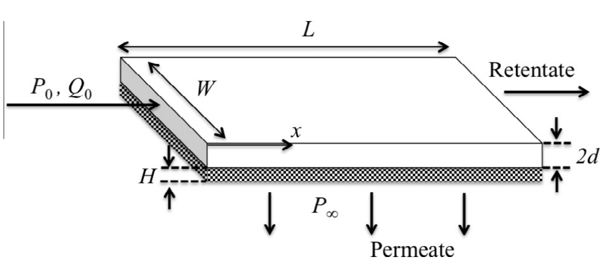

Fig. 1. Schematic of slit domain.

The case $n_w = 2$ of Eq. (2), the continuity equation, can be found in [1]. Eq. (3), Darcy's Law, first appeared in [14]. Derivations of several forms of this equation are in [15]. The Hagen–Poiseuille equation, (4), can be found in [16]. Analytical solutions of Eqs. (2)–(4), also satisfying inflow conditions

$$P(0) = P_0, \; Q(0) = Q_0 \tag{5}$$

can be found, for example, using the method of Laplace transforms. The solutions are:

$$Q(x) = Q_0 \cosh\left(\frac{x}{\lambda}\right) - Q_{ref}\sinh\left(\frac{x}{\lambda}\right) \tag{6}$$

$$P(x) = P_\infty + (P_0 - P_\infty)\cosh\left(\frac{x}{\lambda}\right) - \Delta P_{ref}\sinh\left(\frac{x}{\lambda}\right) \tag{7}$$

$$\varphi(x) = \frac{Q_{ref}}{n_w W \lambda}\cosh\left(\frac{x}{\lambda}\right) - \frac{Q_0}{n_w W \lambda}\sinh\left(\frac{x}{\lambda}\right) \tag{8}$$

where the *design parameters* λ, Q_{ref} and ΔP_{ref} are defined by Eqs. (9)–(11).

$$\lambda = \sqrt{\frac{2d^3 H}{3\; n_w \Gamma}} \tag{9}$$

$$Q_{ref} = \frac{2d^3 W(P_0 - P_\infty)}{3\mu\lambda} \tag{10}$$

$$\Delta P_{ref} = \frac{H Q_0 \mu}{n_w \Gamma \lambda W} \tag{11}$$

2.2. Limiting values for filter length, inlet flow rate and pressure drop

As was done for the cylindrical filter in [11], the exact solutions for the rectangular filter can be used to determine a maximum filter length, L_∞, which depends on λ, Q_0 and Q_{ref}:

$$L_\infty = \begin{cases} \lambda \tanh^{-1}\left(\frac{Q_0}{Q_{ref}}\right) & \text{if } Q_0 < Q_{ref} \\ \lambda \tanh^{-1}\left(\frac{Q_{ref}}{Q_0}\right) & \text{if } Q_0 > Q_{ref} \end{cases} \tag{12}$$