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Model for tracing the path of microparticles in continuous flow microfluidic devices for 2D focusing via standing acoustic waves

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ABSTRACT

An, experimentally validated, two-dimensional dynamic model for tracing the path of microparticles in a microfluidic layered transducer is developed. The model is based on Newton's 2nd law and considers forces due to inertia, gravity, buoyancy, virtual mass and acoustics; it is solved using finite difference method. Microparticles' trajectory consists of transient and steady state phases. All operating and geometric parameters are influential during the transient phase. The final levitation height is independent of the radius and initial vertical location of the microparticle as well as volumetric flow rate; however, dependent on the acoustic energy density and wavelength. There exists a threshold acoustic energy density for levitating microparticles from a specific initial vertical displacement; analytical equation for determining this acoustic energy density is provided.

1. Introduction

Microfluidic devices have found applications in different fields of engineering; many of these applications involve either one or a combination of tasks such as detection, sorting and separation of microparticles. For successfully completing these tasks it is important to focus the microparticles, i.e. arranging randomly distributed microparticles in an orderly fashion; with regards to focusing there are two options, i.e. 2D and 3D focusing [1]. 2D focusing refers to arranging microparticles, which are originally distributed in a random manner, on to single plane while the arrangement of randomly distributed microparticles in a single file (a line of microparticles with each arranged after another) is termed as 3D focusing. Several phenomena including dielectrophoresis, acoustophoresis and hydrodynamics have been adopted by researchers for focusing microparticles [1]. Though 3D focusing of microparticles has been successfully demonstrated by researchers, it is not always necessary for purposes of detection/sorting/separation of the same. Detection/sorting/separation of microparticles can be achieved even with 2D focusing as evidenced from literature [2,3]. The preference for the order of focusing, i.e. 2D or 3D, depends on the phenomena that is employed downstream for purposes of detection/sorting/separation. For example, 3D focusing is a must when detection is carried out using optical means [4,5]; however, 2D focusing has been demonstrated to be sufficient when phenomena such as dielectrophoresis and acoustophoresis are employed for detection [2,3]. Due to the importance of focusing with regards to successfully carrying out detection/sorting/separation in microfluidic devices, it is important to understand the different geometric and operating parameters that influence focusing. Subsequently this article develops a dynamic model for purposes of understanding the influence of different parameters on 2D focusing via standing acoustic waves.

Townsend et al. [6] modelled the trajectory of microparticles in a layered resonator. The model considers the influence of forces due to inertial, drag, gravity, buoyancy and acoustics on the trajectory of microparticles. The influence of virtual mass force and Bassets History force are neglected in this model. In recent years there has been interest in microfluidic devices employing standing surface acoustic waves (SSAW); to this extend researchers have developed analytical equations for predicting the displacement of microparticles in such microfluidic devices. Shi et al. [7] developed a model, by balancing the primary acoustic radiation force and drag force, and in turn realized an analytical solution for the lateral movement of microparticles. Nam et al. [8] improved the model

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Nomenclature

A	area (m^2)
E	energy density ($J\ m^{-3}$)
$\mathbf{F}$	force (N)
H	height (m or μm)
g	acceleration due to gravity ($m\ s^{-2}$)
m	mass (kg)
R	radius (μm or m)
Q	volumetric flow rate ($m^3\ s^{-1}$ or $\mu l\ min^{-1}$)
P	power (W or mW)
t	time (s)
U	resultant velocity of microparticle ($m\ s^{-1}$)
V	volume (m^3)
W	width (m or μm)
$\mathbf{x}$	displacement in the x-direction (m or μm)
Y	initial position of microparticle in the y-direction (m or μm)
$\mathbf{y}$	displacement in the y-direction (m or μm)

Greek Alphabets

α	aspect ratio $\left(\alpha = \frac{W_{ch}}{H_{ch}} \gg 1\right)$
β	compressibility (Pa^{-1})
Δt	time step (s)
λ	wavelength (m or μm)
μ	dynamic viscosity (Pa s)
ν	kinematic viscosity ($m^2\ s^{-1}$)
ρ	density ($kg\ m^{-3}$)
ω	applied frequency ($rad\ s^{-1}$)

Superscripts

$0, 1, 2, \ldots n{-}1, n, n+1$ time step

Subscripts

ac	acoustic
avg	average
ch	microchannel
PAF	primary acoustic radiation
B	buoyancy
D	drag
eq	equivalent
G	gravity
i	input
m	medium
p	microparticle
VM	virtual mass

developed by Shi et al. [7] by including the influence of diffusion and subsequently realized an analytical solution. The forces due to inertia, virtual mass and Bassets History term are neglected in these two models.

In comparison with existing literature the model presented in this article accounts for all forces acting on a microparticle as described by Riley and Maxey [9]. These forces include that due to inertia, gravity, buoyancy, acoustics, virtual mass and Bassets History term. In addition, the model is used for carrying out parametric study; geometric and operating parameters are considered for the parametric study. Moreover, this study is the first to identify the existence of the minimum acoustic energy density, associated with the initial vertical location, necessary for levitating microparticles; an analytical equation for determining this acoustic energy density is provided in this study.

2. Theoretical model

The model developed in this study is specific for microfluidic layered resonators [10]. Fig. 1 provides a schematic of a layered resonator; the device consists of multiple layers. The acoustic transducer is one of the layers and is attached at the bottom of the microdevice. The top layer of the microchannel acts as the reflector. The acoustic wave generated by the acoustic transducer travels towards the reflector and upon striking the wall of the reflector the acoustic wave is reflected back towards the transducer. The acoustic wave originating from the transducer and that reflected from the reflector superimpose to generate a standing acoustic waves inside the microchannel. The height of the microchannel is limited to ½ λ so that it contains just one pressure node and two pressure anti-nodes [10]. The microdevice considered in this article, Fig. 1, operates under continuous flow; the flow is pressure-driven with the sample introduced at the inlet. The microparticles are acted up on by several forces including the acoustic force when transported through the microchannel by the flow. The net force acting on each microparticle levitates the same towards the equilibrium position thereby achieving focusing.

In order to ease the development of the model, without loss of accuracy in describing the underlying physical phenomena, certain assumptions are made and these are listed below.

(a) The microparticle is rigid and spherical;
(b) Stokes flow (creeping flow) exists inside the microchannel; the flow is thus fully developed with velocity varying only along the height of the microchannel;
(c) the radius of the microparticle is much smaller than the wavelength of the acoustic wave;
(d) the path of microparticles is influenced by fluid flow while the fluid flow pattern is not influenced by microparticles;
(e) there are no particle-to-particle as well as particle-to-wall interactions inside the microchannel;
(f) particle Reynolds number is much smaller than unity;
(g) primary acoustic radiation force is the only acoustic force acting on the microparticle; and
(h) no microparticle experiences rotation about its axes during translation inside the microchannel.

As the fluid flow is fully developed, the flow velocity is mathematically described using Eq. (1) [11]. The corresponding average velocity and Reynolds number are provided in Eqs. (2) and (3), respectively. Eq. (4) is the particle Reynolds number.

$$u_m = 6\frac{Q_m}{\alpha H_{ch}^2}\left(\frac{y_{ch}}{H_{ch}} - \frac{y_{ch}^2}{H_{ch}^2}\right) \tag{1}$$

$$u_{m,avg} = \frac{Q_m}{\alpha H_{ch}^2} \tag{2}$$

$$Re_m = \frac{D_{hy} u_{m,avg}}{\nu_m} \tag{3}$$

$$Re_p = \frac{2R_p |u_m|_{y_p} - U_p|}{\nu_m} \tag{4}$$