Separation and Purification Technology 147 (2015) 82–89

Contents lists available at ScienceDirect

Separation and Purification Technology

journal homepage: www.elsevier.com/locate/seppur

Percolation-based model for straining-dominant deep bed filtration

Binbin Ding [a], Chaolin Li [a,*], Xiaoqing Dong [b]

[a] Environmental Science and Engineering Research Center, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, PR China
[b] Department of Environmental Engineering Technology, Shenzhen Institute of Information Technology, Shenzhen 518172, PR China

ARTICLE INFO

Article history:
Received 15 November 2014
Received in revised form 2 April 2015
Accepted 7 April 2015
Available online 23 April 2015

Keywords:
Filtration
Percolation
Porous media
Model

ABSTRACT

A straining-dominant filtration model based on directed bond percolation is proposed. The infinite cluster structure of this model is relevant to filtration properties. Thus, a direct electrifying algorithm is modified for infinite cluster backbone identification, and the geometrical characteristic of the infinite cluster is analyzed. In addition, the relationship between the normalized effluent concentration and infinite cluster is established. The model is validated by comparing the particle concentrations estimated from analyzing the property of filtration cluster of network model with the normalized effluent concentration obtained from filtration process simulation and the experimental data. The results show that normalized effluent concentrations estimated from cluster analysis are consistent with the experimental data and filtration simulated result. Finally, the sensitivity of pore size distribution (PSD) parameters to normalized effluent concentrations estimation is analyzed. Assuming that the PSD is lognormal, the normalized effluent concentrations increase as the location parameter μ increases and scale parameter σ decreases.

1. Introduction

The flow of suspended or colloid particles in porous media is essential to numerous industrial and natural processes [1,2] such as microfiltration [3], packed-bed filtration [4–6], transport of colloidal contaminants (including bacteria and viruses) in ground water [7,8], and the formation of river beds [9]. Therefore, the filtration simulation is important to various industrial applications to optimize the operation conditions and minimize costs. Filtration is often used to remove particles from liquid suspension using porous media. The mechanism of colloid particles removal is interception, which can be classified by advection, diffusion or sedimentation. A special case is straining in pore throats too small for a given colloid particle to pass.

The study of the particle behavior in porous media requires the selection of a model to describe the porous matrix. In the past decade, different models have been developed to simulate the deposition and migration of colloidal particles. Ives [10] first proposed a phenomenological model for deep filtration to predict the reduction in permeability from the analytical solutions. However, many studies [11–13] have revealed that the retained particle concentration estimated by this model is not consistent with the experimental data because without consideration of the straining effects by particle size distribution and pore size distribution (PSD).

Sharma and Yortsos [14,15] proposed a population balance model for deep filtration in which PSD is considered. This model consists of a particle population balance equation and the rates of particle retention and pore blocking caused by the various particle deposition and migration mechanisms. The average particle exclusion rate can be estimated from population balance approaches [16,17] and was successfully used to process data from particulate flow with incomplete pore clogging [17,18]. The contradiction about the model was that the effective length of the capillary regressed from the data of laboratory tests was quite larger than the typical pore diameter [18]. This condition indicates the existence of a special correlation length in the porous medium, which is far larger than the diameter of pore. Thus, a new stochastic model should be constructed to elucidate these observations.

Percolation theory is a stochastic theory applied to porous media [19–21] and closely connected to network models. Network models focus on the effects of pore scale physics, whereas percolation theory highlights the importance of the effects of randomness on macroscopic properties [22], fluid properties, and their interplay. The network model combined with percolation theory is applied to process experimental data [23]. Based on analysis of the feature in the infinite cluster, two power laws are proposed to describe the filtration coefficients close to the percolation threshold and far above the percolation threshold. However, percolation analysis is based on ordinary bond percolation, not the directed bond percolation similar to the filtration process. In addition, the PSD estimation method is defective considering that the relation

* Corresponding author.
E-mail address: lichaolin@hitsz.edu.cn (C. Li).

http://dx.doi.org/10.1016/j.seppur.2015.04.019
1383-5866/

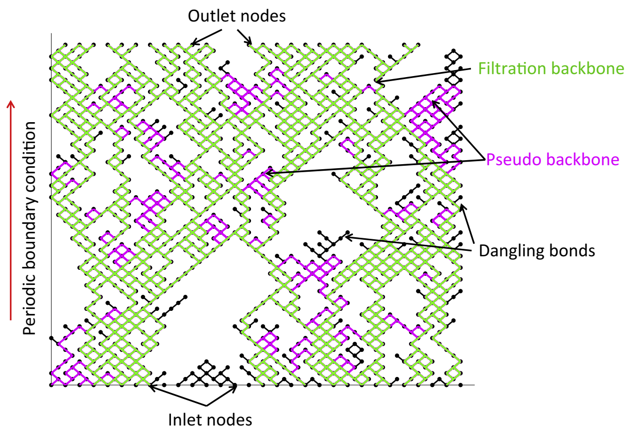

Fig. 1. Schematic representation of a network with line inputs and outputs (color: green, filtration backbone; purple, pseudo backbone; and black, dangling bonds). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

of cluster scale property is based on an assumption of compact packing (three interconnected grains form one pore throat). Therefore, the simulation results are compared with experimental data. The trend of the simulation is similar to that of the experimental data, but the normalized effluent concentrations and the power law exponents estimated by the laboratory tests are inconsistent with those simulated by the 2D network model.

In this study, a model based on directed bond percolation for straining-dominant deep bed filtration is proposed. Numerical solutions for particle concentrations are obtained from filtration process simulation and analysis of the property of the network model filtration cluster. Finally, the normalized effluent concentrations are determined by numerical cluster analysis and compared with experimental data in the literature.

2. Classic model for deep bed filtration

The most commonly used model for deep bed filtration comprise a system of equations containing the continuity equation and particle retention kinetics [24]

$$\begin{cases} \frac{\partial c}{\partial T}+\frac{\partial c}{\partial X}=-\frac{\partial \sigma_r}{\partial T} \\ \frac{\partial \sigma_r}{\partial T}=\lambda(\sigma_r)Lc \end{cases} \tag{1}$$

where $X=\frac{x}{L},\quad T=\frac{1}{L\phi}\int_0^t U(t)dt$.

In Eq. (1), c is the concentration of particles in suspension, σ_r is the concentration of retained particles, λ denotes the filtration coefficient (m^{-1}), and X and T are the dimensionless coordinate and time, respectively. In addition, L is the length of medium, ϕ is the porosity. U indicates Darcy's velocity, and t is time.

The following assumptions are considered: the porous medium is particle-free at the beginning and the injected particle concentration (c_0) is constant. If the filtration coefficient (λ) is constant, and this initial and boundary conditions are applied to the classic model, the analytical solution for the model can be written as

$$c(X,T)=\begin{cases} c_0e^{-\lambda LX}; & X\leqslant T \\ 0; & X>T \end{cases} \tag{2}$$

If the effluent concentration (c_{eff}) is known, the solution to the inverse problem can be easily obtained from [25]

$$\lambda=\frac{1}{L}\ln\left(\frac{c_0}{c_{eff}}\right) \tag{3}$$

You et al. [18,26] presented a microscale model for particle straining; that is, the stochastic parallel tube model (PTM) that describes the suspension flow in porous media. The medium is represented by the model of triangular parallel capillaries alternated with mixing chambers. The analytical model for low-retention filtration is derived, and the steady-state solution is obtained:

$$C_e(r_s)=C_0(r_s)[f_a(r_s)+f_{nl}(r_s)]\exp\left[-f_{ns}(r_s)\frac{L}{l}\right] \tag{4}$$

As shown in Eq. (4), r_s is the injected particle size, f_{ns} is the flow fraction via the inaccessible small pores, f_a+f_{nl} is the fractional flow via large pores, L is the length of column, l is the distance between two chambers, and C_e is the lower outlet concentration. When the particle size reaches the maximum pore size, the outlet concentration vanishes because the flow fraction via the large pores drops to zero. With the PTM model, the PSD is determined by injecting colloidal particle suspensions into engineered porous media with monitored inlet and breakthrough particle concentrations [18].

3. Percolation-based filtration model

3.1. Fundamental statistical parameters

Assuming that the PSD is lognormal, the probability density function of pore throat radius (r_p) is given by

$$f(r_p)=\frac{1}{r_p\sqrt{2\pi}\sigma}e^{\frac{-(\log r_p-\mu)^2}{2\sigma^2}} \tag{5}$$

where μ is the location parameter, while σ is scale parameter. The two parameters μ and σ are, respectively, the mean and standard deviation of the variable's (r_p) natural logarithm.

In the filtration processes with straining as the particle capture mechanism, particles can move from pore to pore until its radius, r_s, is larger than the pore throat radius, r_p. Therefore, the fraction of larger pores (f_l) is a key parameter and can be written in terms of $f(r_p)$ as

$$f_l(r_s)=\int_{r_s}^{\infty}f(r_p)dr_p \tag{6}$$

Assuming that the flow resistance in a single capillary follows Poiseuille's law, the particle flux passing through larger pores as the probability of conducting particle flow can be rewritten as

$$f_l^*(r_s)=\frac{\int_{r_s}^{\infty}r_p^4f(r_p)dr_p}{\int_0^{\infty}r_p^4f(r_p)dr_p} \tag{7}$$

In this study, the PSD follows a lognormal distribution, and the substitution of $f(r_p)$ particle flow through larger pores (f_l^*) can be calculated as follows:

$$f_l^*(r_s)=\frac{\int_{r_s}^{\infty}r_p^4f(r_p)dr_p}{\int_0^{\infty}r_p^4f(r_p)dr_p}=\frac{1+\mathrm{Erf}\left(\frac{\mu+4\sigma^2-\log r_s}{\sqrt{2}\sigma}\right)}{2} \tag{8}$$

Below the percolation threshold, the particle conductivity in the medium is zero. For the bond percolation, f_l^* at the percolation threshold can be calculated as

$$f_c^*(r_s)=\frac{\int_{r_{sc}}^{\infty}r_p^4f(r_p)dr_p}{\int_0^{\infty}r_p^4f(r_p)dr_p}=\frac{1+\mathrm{Erf}\left[\frac{\mu+4\sigma^2-\log r_{sc}}{\sqrt{2}\sigma}\right]}{2} \tag{9}$$

where f_c^* is the flow-biased percolation threshold, and r_{sc} is the particle radius at the threshold.