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Unsaturated hydraulic conductivity modeling for porous media with two fractal regimes

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ABSTRACT

A reliable means to predict the saturation-dependence of the hydraulic conductivity would have important applications and implications across soil science. In our efforts to improve predictive capabilities we apply a bimodal pore size distribution to generate simultaneously the soil water retention curve (SWRC) and the unsaturated hydraulic conductivity K in porous media. Our specific pore size model incorporates two fractal regimes, which we treat within the pore–solid fractal approach. The calculation of the hydraulic conductivity employs critical path analysis from percolation theory, which has already been shown to perform the best overall among models commonly employed. To evaluate the developed piecewise functions, 8 soil samples with different textures, e.g., loam, silt loam, sandy loam and clay are selected. All soils show almost the same cross-over point on both water retention and hydraulic conductivity curves on semi-log plots. We find that the piecewise water retention and unsaturated hydraulic conductivity models fit well the measured data. However, the hydraulic conductivity curves predicted from the water retention data agree relatively well with the measured one just for the first regime and tend to underestimate K in the second. We also compare our results with those obtained from unimodal pore-size distribution reported by Ghanbarian-Alavijeh and Hunt (2012). Comparing the measured data with the unimodal and bimodal models indicates that the bimodal distribution provide somewhat more realistic predictions than the unimodal one. If prediction is sacrificed and we simply try to model K using our results, we find that we can generate a very accurate phenomenological description of K with only a slight change in the values of the fractal dimensionality. Reasons for this discrepancy are discussed.

1. Introduction

A long-cherished goal of soil physics is the ability to predict the volumetric water content (θ) dependence of the hydraulic conductivity, $K(\theta)$, from knowledge of the water retention curve, $\theta(h)$ (where h is tension head). We have recently addressed this problem (Ghanbarian-Alavijeh and Hunt, 2012), showing that application of critical path analysis to a rather simple (monomodal) model, the pore–solid fractal model, generated relatively good predictions, especially compared with other models, e.g., van Genuchten–Mualem (Mualem, 1976; van Genuchten, 1980) in common use. However, we noted some cases where such a simple model of the medium was not realistic enough to capture the structure of the SWRC, which makes the further comparison with experimental values of $K(\theta)$ questionable. In some of these cases the SWRC presented a distinct change in slope at an intermediate water content, indicating that the appropriate model of the medium must at least contain two distribution modes, i.e., be a bimodal distribution. In fact, however, this is not a surprising conclusion as it has already been the subject of discussion elsewhere (Hunt and Gee, 2002).

The disordered structure of soils can be quantified using statistically self-similar fractal models. However, a fractal model is never more than an approximation to the true structure of soil (Crawford et al., 1995). Typical fractal models presented in the literature consider one fractal dimension which scales the hierarchical property of a medium within the range of lower and upper limits (e.g., smallest and largest pore or particle radii). However, porous media may have more than one fractal regime. In the literature, soils which show two or more fractal regimes have already been reported (Bittelli et al., 1999; Hunt and Gee, 2002; Pachepsky et al., 1995; Rieu and Sposito, 1991b; Wu et al., 1993). Thus, piecewise fractal approaches have been developed to model particle-size distributions (Millán et al., 2003) and soil water retention curves (Hunt and Gee, 2002; Millán and González-Posada, 2005; Ojeda et al., 2006; Russell, 2010) of soils with two fractal regimes.

In addition to bimodal power law distributions, bi- and multi-modal approaches have been also applied to log-normal distributions (Kutilek, 2004; Kutilek and Jendele, 2008; Kutilek et al., 2009; Romano et al., 2011) and sigmoidal functions (Coppola, 2000; Durner, 1994; Othmer et al., 1991; Priesack and Durner, 2006; Spohrer et al., 2006; Zhang

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and van Genuchten, 1994) for the purpose of modeling the water retention curve and thus predicting the unsaturated hydraulic conductivity. Othmer et al. (1991) found that the bimodal van Genuchten model could describe soil water retention curve well. Although the bimodal water retention model instead of a unimodal model combined with the Mualem's approach improved unsaturated hydraulic conductivity estimation, Othmer et al. (1991) interpreted the discrepancy between measured and predicted conductivities as either the Mualem (1976) model may not be fully applicable to all types of soils (e.g., fine textures), or existence of trimodal pore size distribution.

Recently, Kutilek et al. (2009) compared different empirical, semi-empirical and physical models in the modeling of soil water retention curve of soils whose pore-size distributions are bimodal. They found that the bimodal log-normal based model did not fit measured water retention data as well as the bimodal sigmoidal function of the van Genuchten (1980) model.

In order to predict flow and transport properties in porous media, several models can be employed. In soil physics the bundle of capillary tubes is the best known among them. This approach, e.g., parallel (Burdine, 1953) or series–parallel (Hoffmann-Riem et al., 1999; Kosugi, 1999; Mualem, 1976) has been widely applied to model hydraulic conductivity of porous media. However, the main drawback of these models as fitted to experimental hydraulic conductivity data is that the exponent used for the power–law tortuosity-correlation correction factor often takes a negative value (Kosugi, 1999; Romano et al., 2011; Schaap and Leij, 2000). This implies that flow paths should be straighter than straight which is physically impossible. In addition, in the general bundle of capillary tubes model (Hoffmann-Riem et al., 1999; Kosugi, 1999), which is constructed of layers of parallel tubes connected in series and included Burdine (1953) and Mualem (1976) models as its special cases, the number of layers is needed to be known a priori. For example, Priesack and Durner (2006) demonstrated that 3 layers may work much better than 2. Thus, physically-based modern methods, such as percolation theory (Hunt and Ewing, 2009; Sahimi, 1994; Stauffer and Aharony, 1992), effective medium approach (Bernasconi, 1974; Kirkpatrick, 1971, 1973), perturbation theory (Rubin, 2003), and renormalization group theory (Bernasconi, 1978; Fisher, 1998; King, 1989) seem to be more promising in the modeling of effective properties of disordered media like soils. Perturbation theory and effective medium approach are applicable and valid only where permeability fluctuations are small in a system (King, 1989). In contrast, when a porous medium is near its percolation threshold, or when a medium has a broad distribution of the hydraulic and/or electrical conductances, the renormalization group and percolation theories are more appropriate (Sahimi, 2011).

Percolation theory is an important approach to quantify the effect of the interconnectivity of a pore space on its flow and transport properties (Sahimi, 2011). It comes in three different types: site, bond, and continuum (Hunt and Ewing, 2009). Perhaps continuum percolation is the most natural type of percolation in which the position of the two components of a random system are not limited to the discrete sites of a regular lattice (Bunde and Havlin, 1996). The most remarkable feature of percolation theory is the existence of a percolation threshold, below which a spreading process (here, flow or conduction) is confined to a finite region (Feder, 1988). However, the value of the percolation threshold, p_c, is known just for special networks, e.g., for the square (cube) lattice, $p_c = 0.5927$ (0.3116) for site percolation and $p_c = 0.50$ (0.2488) for bond percolation (Stauffer and Aharony, 1992). A good approximation of p_c for continuum percolation in natural porous media can be obtained using the idea of residual (stagnant) water content which can be estimated from the soil water retention curve (Ghanbarian-Alavijeh and Hunt, 2012). As an alternative, a morphological technique can be used to estimate the percolation threshold from microtomography and 3D image analysis of porous media (Liu and Regenauer-Lieb, 2011).

Recently, Ghanbarian-Alavijeh and Hunt (2012) used critical path and percolation scaling analysis from percolation theory, combined with the pore–solid fractal (PSF) model (Bird et al., 2000; Perrier et al., 1999), to develop a new model to predict unsaturated hydraulic conductivity from soil water retention curve data. Their results showed that the proposed model predicted hydraulic conductivity more accurately than the Mualem (1976) approach combined with the water retention model of van Genuchten (1980) and PSF (Bird et al., 2000), in particular at low tension heads. Nevertheless, several limitations were noted, among them that a more general treatment to handle different kinds of pore-size distributions was needed. Here we explore one such complication.

The objective of this study is therefore to extend the Ghanbarian-Alavijeh and Hunt (2012) approach to address the modeling of soil water retention and unsaturated hydraulic conductivity of soils incorporating two fractal regimes. Following Ghanbarian-Alavijeh and Hunt (2012), we apply critical path analysis to model hydraulic conductivity at different saturations. The theory developed here treats two different ranges of pore size where these two regimes can be attributed separately to textural (within soil aggregates) and structural pores (between aggregates) or two different textural pores. We have the capacity to treat the two regimes either as interpenetrating, or as independent in the sense of dual porosity models. The latter theoretical technique was developed in Hunt and Ewing (2009) within the theoretical constraints prescribed by percolation theory, and could be logically applied here in some of the soils investigated. Results from the two different approaches appear in the context of the present percolation-based calculations to be equivalent, in contrast to the situation with the capillary bundle models.

2. Theory

2.1. Soil water retention curve

We assume that the pore-size distribution of soils follows the pore–solid fractal (PSF) approach proposed by Perrier et al. (1999). This model combined with percolation theory has been successfully applied to model unsaturated hydraulic conductivity of soils with different textures (Ghanbarian-Alavijeh and Hunt, 2012). The continuous probability density function, $W(r)$, of pores consistent with the Hunt and Gee (2002) analogy would be:

$$W(r) = \beta \frac{3-D}{r_{\max}^{3-D}} r^{-1-D}, \quad r_{\min} < r < r_{\max} \tag{1}$$

where $\beta = p/(p+s)$ in which p and s are the pore and solid fractions (Perrier et al., 1999), respectively, D is the pore-solid interface fractal dimension, r is the pore radius ($r \propto 1/h$ where h is the tension head), and $r_{\min}$ and $r_{\max}$ are the smallest and largest pore radii, respectively.

Fig. 1 shows the scheme of a soil water retention curve with two fractal regimes. The first regime covers mostly the large (frequently structural) pores, and the second regime includes the small (textural) pores. The water content at the cross-over point is denoted by θ_x which is equal to the porosity of the second regime ϕ_2.

In a porous medium having a probability density function that scales with two different regimes, e.g., D_1 and D_2 (Fig. 1), the total porosity may be found by integrating $r^3 W(r)$ between $r_{\min}$ and $r_{\max}$ to obtain

$$\begin{aligned}\phi = \phi_1 + \phi_2 &= \beta_1 \frac{3-D_1}{r_{\max}^{3-D_1}} \int_{r_x}^{r_{\max}} r^3 r^{-1-D_1} dr + \beta_2 \frac{3-D_2}{r_x^{3-D_2}} \int_{r_{\min}}^{r_x} r^3 r^{-1-D_2} dr \\ &= \beta_1 \left[1 - \left(\frac{r_x}{r_{\max}}\right)^{3-D_1}\right] + \beta_2 \left[1 - \left(\frac{r_{\min}}{r_x}\right)^{3-D_2}\right]\end{aligned} \tag{2}$$

where r_x is the pore radius at the cross-over point where the fractal behavior of the medium changes from regime 1 to regime 2, and D_1 and D_2 are the pore–solid interface fractal dimension of the first and second regimes, respectively.