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Numerical modeling of subsidence in saturated porous media: A mass conservative method

Roza Asadi [a,*], Behzad Ataie-Ashtiani [a,b]

[a] Department of Civil Engineering, Sharif University of Technology, PO Box 11155-9313, Tehran, Iran
[b] National Centre for Groundwater Research & Training and School of the Environment, Flinders University, GPO Box 2100, Adelaide, South Australia 5001, Australia

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ABSTRACT

In this paper, a second order accurate cell-centered finite volume method (FVM) is coupled with a finite element method (FEM) to solve the deformation of a saturated porous layer based on Biot's consolidation model. The proposed numerical technique is applied to the fully unstructured triangular grids to simulate actual geological formations. To reconstruct the pressure gradient at control volume faces, the diamond scheme is implemented as a multipoint flux approximation method. Also the least square algorithm is used to interpolate pressure at the vertices from the cell-center values. The stability of this numerical model is studied in comparison to the different FEMs through various examples. It is shown that, although the Taylor-Hood FEM has been introduced as a remedy for violation of the inf-sup condition, it does not entirely remove the non-physical oscillations. Contrary to the linear and Taylor-Hood FEMs, the proposed discretization model provides monotonic solution without imposing any restriction on the mesh or time step size. Compared to the mixed FEM, the method achieves local mass balance with fewer degrees of freedom. To couple the flow and mechanical sub-problems, the fixed-stress operator split is implemented as an iterative sequential method, due to its unconditional stability, accuracy and high rate of convergence. The accuracy of the proposed model is verified via a range of examples including analytical and numerical solutions. The performance of this methodology is assessed through modeling of subsidence in an aquifer-interbed system. This problem illustrates the capability of the model in providing stable solution in heterogeneous domains with complicated shapes.

1. Introduction

Groundwater is a vital natural resource throughout the world especially in arid and semi-arid regions. Rapidly growing demands for water in agricultural, industrial and municipal applications, causes overexploitation of water supply from aquifers. The resulting groundwater depletion is associated with adverse geo-environmental impacts. The major worldwide hazard originated from extensive pumping is the human-induced subsidence, which is referred as one of the main water-related disasters in IHP-VIII (2014–2021) (International Hydrological Programme-eighth phase) (Gambolati and Teatini, 2015; Donoso et al., 2013). Especially in coastal areas, the impact of subsidence on the flood risk and land inundation is a major concern (Erkens and Sutanudjaja, 2015). For example in the Mekong river delta (Vietnam) with the area of 55,000 square kilometers, the subsidence rate of 1–4.7 cm/year, can lead to about one meter of sinking by midcentury which can affect the life of about 20 million people (Schmidt, 2015).

The conventional model widely used to simulate subsidence problems is based on Terzaghi's theory, taking into consideration the pore compressibility parameter in the diffusion flow equation to describe deformability of the porous medium (Terzaghi, 1943). Although this one dimensional consolidation theory is well suited for the cases where horizontal to vertical strain ratio tends to zero, it has limitations on modeling complex features of fluid flow through compressible porous media (Gutierrez and Lewis, 2002). Such an example is the well-known Mandel-Cryer effect (Gutierrez and Lewis, 2002; Mandel, 1953; Cryer, 1963) which can be explained by Biot's theory (Biot, 1941). Indeed, the uncoupled flow (diffusion) equation, which is applied for addressing many environmental issues, is not adequate to model the dynamic behavior of groundwater flow in hydrological systems that concern fluid-soil interaction.

Since the analytical solutions derived for Biot's differential equations are restricted to simple cases, the numerical methods

* Corresponding author.
E-mail addresses: roza_asadi@mehr.sharif.ir (R. Asadi), ataie@sharif.edu (B. Ataie-Ashtiani).

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provide powerful tools to deal with coupled problems especially in realistic applications. A commonly used numerical technique for spatial discretization of the geomechanical problems is the finite element method (FEM) (Gutierrez and Lewis, 2002; Zienkiewicz et al., 1999; Lewis and Schrefler, 1998; Ferronato et al., 2001). Despite the flexibility of this technique in modeling complex geometries, the standard procedure suffers from numerical oscillations in the time-dependent poroelasticity problems (Vermeer and Verruijt, 1981; Murad and Loula, 1992; Kim, 2010; Ferronato et al., 2010). These nonphysical oscillations occur for two reasons: first, the violation of the inf-sup condition and second, the discontinuity in the derivative of pressure at the interface between two layers with different permeabilities and also at the drainage boundary (Vermeer and Verruijt, 1981; Murad and Loula, 1992; Kim, 2010). Indeed, to obtain the variational formulation of the Biot's consolidation model in the FEM, two function spaces are considered for the pore pressure and displacement fields. It is shown and proven that the stability of the solution of this variational form depends on the inf-sup or LBB (Ladyzenskaja-Babuska-Brezzi) condition (Murad and Loula, 1992, 1994). In this condition, for any pair of functions belongs to these two spaces, the norm of bilinear form of this pair divided by the corresponding norms should be bounded below by the constant which is independent of mesh size (Babuska, 1973; Brezzi, 1974). This well-known condition which is accepted for providing the well posed discrete Galerkin approximation, does not necessarily lead to stable solutions (Vermeer and Verruijt, 1981; Murad and Loula, 1992; Rodrigo et al., 2016). For this reason the stabilized finite element methods have been proposed to eliminate such instabilities (e.g., Wan, 2002; Wan et al., 2003; Truty and Zimmermann, 2006; White and Borja, 2008; Rodrigo et al., 2016). However, this class of discretization methods is not locally conservative (Wan, 2002) and appropriate stabilization term due to the physics and properties of the problem is needed.

In order to overcome the stability problems and ensure local mass conservation in flow domain, the classes of mixed FEM and FVM are presented in recent years for the saturated and multiphase flow problems, (e.g., Phillips and Wheeler, 2007a,b; Jha and Juanes, 2007; Ferronato et al., 2010; Kim, 2010; Caviedes-Voullième et al., 2013; Manzini and Ferraris, 2004; Asadi et al., 2014; Asadi and Ataie-Ashtiani, 2015). Due to the huge computational cost of the mixed FEMs, FVM has been developed in this study to simulate the flow equation. Also the complex geometric domains are treated through the fully unstructured triangular grids, which are the most general and flexible type of mesh for describing complicated geological formations (Lee et al., 2002). Since in this meshing method, the line connecting two adjacent elements centers is not generally orthogonal to the corresponding face, the multipoint flux approximation (MPFA) methods are required to estimate the inter-element flux with higher accuracy. Here, the diamond scheme is implemented as a MPFA method in the framework of cell-centered finite volumes (CC-FV) (Coudière et al., 1999; Coudière and Villedieu, 2000; Bertolazzi and Manzini, 2005, 2004; Manzini and Ferraris, 2004; Bevilacqua et al., 2011). In this approach, by using the least square method to interpolate pressure at the vertices from the cell-center values, the second order of accuracy can be achieved, as investigated in Bertolazzi and Manzini (2004) and Manzini and Ferraris (2004). To complete the spatial discretization of the coupled system, the finite element approach is employed to formulate the mechanical equation (Gutierrez and Lewis, 2002; Zienkiewicz et al., 1999; Lewis and Schrefler, 1998; Gambolati et al., 2001). This combined discretization algorithm (CCFV-FE) benefits from stability and local conservation properties as well as accommodating complex geometries on fully unstructured grids.

To couple the flow and mechanical sub-problems, the iterative sequential method of fixed-stress is implemented. This operator split has been successfully applied to the multi-dimensional coupled systems involving saturated and multiphase flow in elastic, elastoplastic and faulted domains (Kim et al., 2011a, 2011b; Kim, 2010; Jha and Juanes, 2014). Asadi et al. (2014) compared different sequential strategies with various degrees of coupling and demonstrated the superior performance of this method in comparison with other staggered algorithms. This algorithm has successfully been implemented to the different forms of the coupled multiphase flow and geotechnical deformation by Asadi and Ataie-Ashtiani (2015) to reduce CPU time and values of errors. Due to the unconditional stability (Kim et al., 2011a; Kim, 2010), accuracy and higher rate of convergence of this scheme in comparison to other sequential methods (Asadi et al., 2014), it is employed in this study.

The CCFV-FE method proposed in this study is validated and examined against analytical and numerical solutions presented in literature. In addition, to show the capability of the model in tackling land subsidence problems in heterogeneous domains with complicated shapes, the aquifer with a lens-shaped interbed is simulated.

The objective of this study is to develop and validate the coupled flow-geomechanical model with the aim to ensure stability, yield local mass conservation and accommodate complex geometries based on unstructured grids. Moreover, the solver applied in this method is a partitioned solution procedure, which reduces the required memory and utilizes single computational grid for both the flow and mechanical simulators. The proposed hybrid FV-FEM has been compared to other numerical discretization methods in terms of stability, accuracy and computational cost and the advantages of this scheme have been discussed.

2. Mathematical formulation

Based on Biot's theory for consolidation of fluid-saturated porous media, the coupled model of flow and mechanics reads (Biot and Willis, 1957; Lewis and Schrefler, 1998):

$$\frac{\partial}{\partial t}\left(\frac{1}{M}p + \alpha\nabla\cdot\mathbf{u}\right) + \mathrm{div}\mathbf{v} = f \tag{1}$$

$$\mathrm{div}\hat{\boldsymbol{\sigma}} + \mathbf{b} = 0 \tag{2}$$

In the above system of partial differential equations, Eq. (1) defines the conservation of the fluid mass in a deformable porous media in which t is time, p denotes the fluid pressure and $\mathbf{u}$ is the solid displacement vector. M is the Biot modulus which can be defined as $\left(\frac{\phi}{K_f} + \frac{\alpha-\phi}{K_s}\right)^{-1}$, $\mathbf{v} = \frac{\overline{\mathbf{K}}}{\rho g}\nabla p$ represents the Darcy's velocity and α is the Biot coefficient which is equal to $1 - \frac{K_T}{K_s}$. Also ϕ is the porosity, K_f, K_s and K_T are the bulk modulus of the fluid, solid grain and the porous medium, respectively. The term $\overline{\mathbf{K}}$ refers to the hydraulic conductivity tensor, ρ is the fluid density, g is the gravitational acceleration and f is a source/sink term.

Under the quasi-static assumption, the equilibrium equation of the porous medium can be expressed by using Eq. (2) in which $\hat{\boldsymbol{\sigma}}$ is the total stress tensor and $\mathbf{b}$ denotes the body force. Through the concept of Terzaghi's effective stress, the stress relation for the solid phase is (Terzaghi, 1943):

$$\hat{\boldsymbol{\sigma}} = \widehat{\boldsymbol{\sigma}'} - \alpha p\mathbf{I} \tag{3}$$

where $\widehat{\boldsymbol{\sigma}'}$ is the effective stress tensor and $\mathbf{I}$ indicates the Kronecker delta tensor. To complete the mathematical model, the constitutive relationship between the effective stress and strain is assumed to be linear. In order to solve this mathematical model, two sets of boundary conditions according to the flow and mechanical equations are required as follows: