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## Research papers

# A mathematically continuous model for describing the hydraulic properties of unsaturated porous media over the entire range of matric suctions

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## ABSTRACT

Recent studies suggest that water flow in unsaturated porous media extends beyond the commonly known capillary-driven regime into the film regime. There is a need to develop the unsaturated hydraulic properties over the entire range of matric suctions to capture both flow regimes. In this study, Fredlund and Xing model is modified to represent the soil water retention curve from saturation to oven dryness. The new function is mathematically differentiable. The hydraulic conductivity function is composed of the capillary-driven term and film associated term, which is easy to apply. The new model has capacity to represent the bimodal hydraulic properties that are often present in structured and aggregated soils. Testing with the published data of sixteen soils shows good performance for both the water retention curve and the hydraulic conductivity function. For most soils, the new model results in a better agreement with observations than a published model. The result also indicates a possibility to improve the previously published film-associated hydraulic conductivity function.

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## 1. Introduction

The unsaturated zone supports terrestrial ecosystems. It lies between surface and groundwater systems, influencing groundwater recharge and quality. Thus modeling water flow and solute transport in the unsaturated zone is always an important research subject in hydrological and ecological studies. Solutions to the Richards' (1931) equation, the most widely used model for simulating water flow and associated solute transport in variably saturated media, require the soil water retention curve (SWRC)  $h(\theta)$  and the hydraulic conductivity curve  $K(h)$ .

The commonly used SWRC functions (e.g., Brooks and Corey, 1964; van Genuchten, 1980) and soil hydraulic conductivity functions (e.g., Mualem, 1976a) are based on the capillary theory (Mualem, 1976a; Tuller and Or, 2001). This theory conceptualizes pore space into a bundle of cylindrical capillaries (Millington and Quirk, 1961; Mualem, 1976a). Water is supposed to move only when the capillary pendular rings are continuous in the pores

(Tokunaga, 2009), with an assumption that the meniscus of water is axially symmetric or nearly so (Orr et al., 1975; Bear et al., 2011). The water content at the stage with broken pendular rings is defined as the residual water content ( $\theta_r$ ), representing the moisture beyond the capillary influence. This water is assumed to be hydraulically immobile according to the capillary theory.

However, previous studies have found that the conventional SWRCs and soil hydraulic conductivity functions often do not accurately simulate flow processes that occur under conditions of low soil water contents (e.g., Rossi and Nimmo, 1994; Silva and Grifoll, 2007). Water moving along thin liquid films on the particle surfaces can be significant under dry conditions (Li and Wardlaw, 1986; Lenormand, 1990; Toledo et al., 1990; Tuller et al., 1999; Tuller and Or, 2001; Wang et al., 2013), which is, however, neglected in the commonly used soil hydraulic models (Lebeau and Konrad, 2010). The impact from three-dimensional pore network (e.g., pore connectivity and air entrapment) further complicates unsaturated water flow modeling, which is not considered here.

To describe the soil hydraulic properties from saturation to oven dryness, one needs to extend the soil hydraulic models from the capillary-dominated suctions to film-dominated suction range.

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## Nomenclature

## Notation

$A$	the specific perimeter per unit area of the porous media ( $L L^{-2}$ )	$l$	capillary model parameter
$b$	the parameter vector used for optimization	$l_1$	fitted parameter
$\alpha$	fitted parameter ( $L^{-1}$ )	$\lambda$	grain diameter (L)
$\beta$	capillary model parameter	$m$	fitted parameter
$C(h)$	correction function in the Fredlund and Xing (1994) model	$n$	fitted parameter
$c$	scaling factor (=0.01)	$n_K$	the number of conductivity data used for optimization
$dh/dx$	the matric suction gradient	$n_o$	the number of retention data used for optimization
$e$	is Euler's number (the $e$ constant)	$\mu$	viscosity of the fluid ( $M L^{-1} T^{-1}$ )
$f$	film thickness (L)	$\Pi(h)$	the unscaled capillary saturation in the VG model
$f_r$	film thickness at $S_r$ (L)	$\Pi_o$	the capillary saturation at the suction of $h_o$ in the VG model
$f(S), f(1)$ and $f(y)$	the integrated functions of the capillary model	$Q_c$	the specific volumetric flux associated with the capillary force ( $L T^{-1}$ )
$F(y)$	the integrated part of the capillary model in the EMFX model	$Q_f$	the specific volumetric flux associated with the film force ( $L T^{-1}$ )
$F_{VG}(y)$	the integrated part of the capillary model in the VG model	$\rho$	density of the fluid ( $M L^{-3}$ )
$\Gamma(h)$	correction function in the EMFX model	RMSE $_{\theta}$	root mean squared error for water content
$\Phi(b)$	objective function for optimization	RMSE $_{\log K_r}$	root mean squared error for log-scale conductivity
$g$	the acceleration due to gravity ( $L T^{-2}$ )	$S$	saturation degree
$\theta$	volumetric water content ( $L^3 L^{-3}$ )	$S_c$	the effective saturation where the capillary force dominates
$\theta_r$	residual water content ( $L^3 L^{-3}$ )	$S_r$	the saturation degree at the residual water content
$\theta_f$	water content associated with the film force ( $L^3 L^{-3}$ )	$S_f$	the film saturation degree
$\theta_s$	saturated water content ( $L^3 L^{-3}$ )	$\tau$	exponent in the film conductivity model
$\hat{\theta}_i$	model estimated water content ( $L^3 L^{-3}$ )	$v_f$	the film thickness averaged film velocity ( $L T^{-1}$ )
$h$	matric suction (L)	$w$	weight factor for the conductivity in the PDI model
$h_r$	matric suction corresponding to the residual water content (L)	$w_{\theta}$	weight for the water content
$h_o$	matric suction where water content is zero (L)	$w_k$	weight for the conductivity data
$h_a$	the critical matric suction in the PDI model, equals $1/\alpha$ (L)	$X_m$	scaling factor for the film water retention curve in the PDI model
$h(S)$	matric suction as a function of $S$	$y$	variable used for deriving the capillary conductivity, equals $S^{1/m}$
$k$	capillary model parameter	$\delta$	surface tension ( $N L^{-1}$ )
$K$	the total hydraulic conductivity ( $L T^{-1}$ )	SWRC	soil water retention curve
$K_c$	capillary conductivity ( $L T^{-1}$ )	FX model	the model developed by Fredlund and Xing (1994)
$K_f$	film conductivity ( $L T^{-1}$ )	EMFX model	the proposed model as an extension and modification of the FX model
$K_f(S)$	film conductivity as a function of saturation ( $L T^{-1}$ )	PDI model	the model developed by Peters (2013) and improved by Iden and Durner (2014)
$K_f^r$	film conductivity at $S_r$ ( $L T^{-1}$ )	VG model	the model developed by van Genuchten (1980)
$\hat{K}_i$	model estimated conductivity values ( $L T^{-1}$ )		
$K_s$	saturated hydraulic conductivity ( $L T^{-1}$ )		

Depending whether the van der Waals forces or the electrical double-layer forces are the dominant contribution to film thickness, two hydraulic conductivity models were developed to simulate film flow by Tuller and Or (2001) and Tokunaga (2009), respectively. The model proposed by Tuller and Or (2001) accounts for both capillary and film forces. This model must be used in conjunction with the water retention model of Or and Tuller (1999), which often fails to fit experimental data in the intermediate saturation range (Lebeau and Konrad, 2010). Additionally, the electrical double-layer force is considered more important at the intermediate values of matric potential (Tokunaga, 2011). By combining an existing capillary conductivity model (e.g., van Genuchten (1980) model) with Tokunaga (2009) film model that accounts for electrical double-layer forces, a series of models have been developed to describe the soil hydraulic conductivity from saturation to oven dryness (Lebeau and Konrad, 2010; Zhang, 2011; Peters, 2013). Two problems exist in these models. First, the film conductivity model presented by Tokunaga (2009) relies on the smooth sphere grain assumption, its validation for different soils needs to be

examined. Secondly, for the SWRCs, these models often have the “mathematical discontinuity” problem (Iden and Durner, 2014). This problem results from that the water content is divided into the capillary and film components. The SWRCs that account for film flow (e.g., Campbell and Shiozawa (1992) model) are not differentiable over the entire range of suctions. This can lead to a numerical problem (Iden and Durner, 2014). A smoothing strategy may solve this problem, but it makes the model mathematically more complex.

Fredlund and Xing (1994) adapted a different approach. They describe the SWRC from saturation to oven dryness without partitioning water into the capillary and adsorptive components, so the equation is differentiable over the entire matric suction range. However, there is no closed-form solution for the hydraulic conductivity function, and film flow contribution is not considered.

In this paper, a modification of Fredlund and Xing (1994) model is provided to describe the SWRC over the suction range from saturation to oven dryness. The model is mathematically continuous and easy to use. Based on the modified model, an approximately

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