Journal of Hydrology 541 (2016) 928–934

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

Solute dispersion in a semi-infinite aquifer with specified concentration along an arbitrary plane source

Mritunjay Kumar Singh *, Ayan Chatterjee

Department of Applied Mathematics, Indian School of Mines, Dhanbad 826004, India

ARTICLE INFO

Article history:
Received 20 May 2016
Received in revised form 20 July 2016
Accepted 1 August 2016
Available online 3 August 2016
This manuscript was handled by P. Kitanidis, Editor-in-Chief, with the assistance of Todd C. Rasmussen, Associate Editor

Keywords:
Analytical and numerical solutions
Non point source
Contamination
Aquifer

ABSTRACT

Mathematical modeling is carried out to investigate the behavior of solute transport in groundwater with non-point sources of contamination i.e., plane and line sources. Generalized form of plane is considered to study the most general case for non-point sources. Using the hyper-plane concept, the closed-form solution is derived for line sources in two-dimensions as well as the point source in one-dimension as special cases of the general plane source contamination problem. The domain is considered semi-infinite and the velocity of groundwater is in the positive direction of the axes. The three-dimensional advection-dispersion equation (ADE) with non-point source of contamination is solved analytically using the Laplace transform technique. The obtained solution is validated numerically using the finite difference technique. The proposed general solution of the three-dimensional ADE may be of interest to researchers working in surface water and vadose zone hydrology areas as well.

1. Introduction

Groundwater is the primary source of drinking water in many parts of the world. Groundwater contamination through various sources like point and non-point sources are becoming a major problem in India and many others countries. Various wastes from industries, fertilizers, pesticides and many other substances mix with groundwater which leads to contamination of drinking water. In the last 50 years, pesticides have been used extensively to improve the quality of food for the growing population. In porous media and groundwater, modeling studies with point source contamination have been carried out to estimate the contaminant concentration. However, in reality non-point sources of contamination like industrial tanks and open mine pits exist. The easiest way to deal with the non-point source is to consider a plane source in three-dimensions and a line source in two-dimensions.

In the past few decades, a majority of the modeling studies associated with groundwater contamination focused on point sources. Brooks (1960) discussed diffusion of sewage effluent in an ocean current. Yates (1992) derived one-dimensional analytical solution to a generalized ADE with an exponentially varied dispersivity and constant concentration or flux inlet boundary conditions in a semi-infinite domain, using the Laplace transform technique. Logan (1996) presented solute transport in porous media with scale-dependent dispersion and periodic boundary conditions which was an extension of the work of Yates (1992). Aral and Liao (1996) obtained a general analytical solution of the two-dimensional solute transport equation with time-dependent dispersion coefficient for an infinite domain aquifer. Batu (1996) explored a generalized three-dimensional analytic solute transport model for multiple sources. Widespread contamination of U.S. water resources were found by the U.S. Geological Survey. In particular, Robert et al. (1999) presented that more than 95% of samples collected from streams, and almost 50% of samples collected from wells, contained at least one pesticide. Diwa et al. (2001) discussed one-dimensional simulation of solute transfer in saturated-unsaturated porous media using the discontinuous finite element method. All pesticides in groundwater, and most residues present in surface water bodies enter via the soil which is considered as a plane-source for groundwater contamination. Rubin and Atkinson (2001) discussed various properties of environmental fluid flow modeling in environmental fluid mechanics. Younes (2003) presented modeling of the multi-dimensional fluid flow and heat or mass transport in porous media. Guyonnet and Neville (2004) discussed dimensionless analysis of analytical solution for three-dimensional solute transport in groundwater. Sander and Braddock (2005) explored analytical solution to the transient, unsaturated transport of water and contaminants through horizon-

* Corresponding author.
E-mail addresses: drmks29@rediffmail.com (M.K. Singh), ayan@am.ism.ac.in (A. Chatterjee).

http://dx.doi.org/10.1016/j.jhydrol.2016.08.003

Nomenclature

Symbols	Descriptions
C	contaminant concentration
x, y, z	space variables
t	time variable
D_x, D_y, D_z	dispersion coefficients along x, y, z axes respectively
u_x, u_y, u_z	seepage velocities of the solute particle along x, y, z axes respectively
c_0	concentration at the source
a_1, b_1, e_1, f, k_1 and k_2	arbitrary constants
$g(t)$	function of time
X	new space variable
T	new time variable
$K(X, T)$	function dependent on X and T
$u_{x_0}, u_{y_0}, u_{z_0}$	initial seepage velocities along x, y, z axes respectively
$D_{x_0}, D_{y_0}, D_{z_0}$	initial dispersion coefficients along x, y, z axes respectively

tal porous media. Smedt (2006) derived an analytical solution for transport of decaying solutes in rivers with transient storage. Singh et al. (2008) presented longitudinal dispersion with time-dependent source concentration along unsteady groundwater flow in a semi-infinite aquifer. Srinivasan and Clement (2008) discussed analytical solution for sequentially coupled one-dimensional reactive transport problem. Singh et al. (2010a) presented one- and two-dimensional analytical solutions using Laplace and Hankel transform techniques respectively with suitable initial and boundary conditions. Zhan et al. (2009) discussed an analytical solution of two-dimensional solute transport in an aquifer-aquitard system. Analytical solutions of one-dimensional solute transport along and against time-dependent/scale dependent point source contamination in homogeneous/heterogeneous semi-infinite aquifer were explored (Singh et al., 2010b; Kumar et al., 2010; Singh and Kumari, 2014; Singh and Das, 2015). The generalized analytical solutions for advection-dispersion equation in finite spatial domain with arbitrary time-dependent inlet boundary condition were presented (Chen and Liu, 2011; Chen et al., 2012). Vasquez et al. (2013) introduced the modeling flow and reactive transport to explain mineral zoning in the Atacama salt flat aquifer.

Line, horizontal and vertical plane sources were also considered by some of the researchers to model the non-point groundwater contamination problems (Ogata, 1970; Javandel et al., 1984). Ogata (1970) and Javandel et al. (1984) presented an analytical solution for line sources. Land filled and land treatment facilities were more accurately modeled using plane sources conditions. Green's function technique solution was used to model the dispersive transport in groundwater for a variety of source conditions except horizontal plane sources (Codell and Schreiber, 1977; Yeh, 1981). Galya (1987) extended Codell and Schreiber (1977) and Yeh's (1981) work and solved the problem for horizontal plane source using Green's function technique. Sayre (1973) considered an infinite domain with a continuous line source at $x = 0$ and $y = 0$ from a non-reactive constituent with constant release rate. Van Genuchten et al. (2013) considered one-dimensional and multi-dimensional solutions for the standard equilibrium ADE with and without terms accounting for zero-order production and first-order decay. They considered various types of source conditions like line and plane sources of either constant or time-dependent concentration.

Depending upon the geometry it is clear that the plane sources may not always be vertical or horizontal, it can be a general plane which is the most appropriate case of plane sources in groundwater contamination. In this study a general plane source problem is considered to evaluate the impact of non-point sources in groundwater contamination problems. The importance of plane source contamination is quite significant for modeling real life situations as it is not always possible to model a physical system using point source contamination. For example, the effect of the pesticides used during cultivation can not be truly represented by using point source contamination. Therefore, non-point source contamination as a plane source is considered in this study. The solution of the three-dimensional ADE may be of interest to researchers working in surface water and vadose zone hydrology areas as well. From the three-dimensional solution for plane source, one can derive the two-dimensional solution for a line source and one-dimensional solution for a point source. The Laplace transform and finite-difference techniques are used to solve the system analytically and numerically respectively. Constant type source is taken into consideration throughout the plane.

2. Mathematical formulation for plane source in three-dimensions

The problem is formulated mathematically as the plane source in three-dimensions. The domain is considered semi-infinite and the aquifer is initially contaminant free. The contamination flux is zero when $x \longrightarrow \infty, y \longrightarrow \infty$ and $z \longrightarrow \infty$. Van Genuchte and Parker (1984) discussed the zero flux condition which was considered at the semi-infinite part of the boundary ($x \longrightarrow \infty, y \longrightarrow \infty$). The geometry of the problem is shown in Fig. 1.

The general plane source problem is formulated with the conditions that initially the aquifer is contaminant free and the flux at semi-infinite boundary is zero. The constant contamination (c_0) is considered through the general plane $a_1x + b_1y + e_1z = f$ as the boundary condition, where a_1, b_1, e_1 and f are arbitrary constants. Three-dimensional ADE equation is used to model the system mathematically and it can be written as follows:

$$\frac{\partial C}{\partial t} = D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2} + D_z \frac{\partial^2 C}{\partial z^2} - u_x \frac{\partial C}{\partial x} - u_y \frac{\partial C}{\partial y} - u_z \frac{\partial C}{\partial z} \quad (1)$$

where $C(x, y, z, t)$ is solute concentration; x, y, z denote the spatial co-ordinates; t is time; D_x, D_y, D_z are the dispersion coefficients and u_x, u_y, u_z are seepage velocities along the co-ordinate axes x, y, z respectively.

Initial and boundary conditions are as follows:

$$C(x, y, z, t) = 0, \quad t = 0, \quad x, y, z \geqslant 0, \quad (2)$$

$$C(x, y, z, t) = c_0, \quad a_1x + b_1y + e_1z = f, \quad t > 0, \quad (3)$$

$$\frac{\partial C}{\partial x} = 0; \quad \frac{\partial C}{\partial y} = 0; \quad \frac{\partial C}{\partial z} = 0; \quad x, y, z \to \infty \quad (4)$$

3. Analytical solution

To understand contaminant transport in the presence of a plane source, the three-dimensional ADE given in Eqs. (1)–(4) is solved analytically using Laplace transform technique. The dispersion theory proposed by Freeze and Cherry (1979), the dispersion is directly proportional to the power of the seepage velocity where