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A study of non-linearity in rainfall-runoff response using 120 UK catchments

Simon A. Mathias [a,*], Neil McIntyre [b], Rachel H. Oughton [a]

[a] Department of Earth Sciences, Durham University, Durham, UK
[b] Centre for Water in the Minerals Industry, Sustainable Minerals Institute, The University of Queensland, Brisbane, Australia

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ABSTRACT

This study presents a catchment characteristic sensitivity analysis concerning the non-linearity of rainfall-runoff response in 120 UK catchments. Two approaches were adopted. The first approach involved, for each catchment, regression of a power-law to flow rate gradient data for recession events only. This approach was referred to as the recession analysis (RA). The second approach involved calibrating a rainfall-runoff model to the full data set (both recession and non-recession events). The rainfall-runoff model was developed by combining a power-law streamflow routing function with a one parameter probability distributed model (PDM) for soil moisture accounting. This approach was referred to as the rainfall-runoff model (RM). Step-wise linear regression was used to derive regionalization equations for the three parameters. An advantage of the RM approach is that it utilizes much more of the observed data. Results from the RM approach suggest that catchments with high base-flow and low annual precipitation tend to exhibit greater non-linearity in rainfall-runoff response. In contrast, the results from the RA approach suggest that non-linearity is linked to low evaporative demand. The difference in results is attributed to the aggregation of storm-flow and base-flow into a single system giving rise to a seemingly more non-linear response when applying the RM approach to catchments that exhibit a strongly dual storm-flow base-flow response. The study also highlights the value and limitations in a regionlization context of aggregating storm-flow and base-flow pathways into a single non-linear routing function.

1. Introduction

Rainfall-runoff modeling has long been recognized as an important methodology for improving our hydrological understanding of river catchments. Rainfall-runoff models are typically used to forecast river flow data for a given set of precipitation and potential evapotranspiration data (Wagener et al., 2001). Such models often have unknown model parameters that can be obtained by calibrating the models to observed river flow data (Wagener et al., 2001). For ungauged catchments (where no record of flow observations exist), model parameters can be estimated using regionalization relationships (Young, 2006).

Regionalization relationships are typically obtained by calibrating a rainfall-runoff model to multiple catchments and developing statistical relationships between the model parameters and non-flow data dependent parameters, often referred to as catchment characteristics (Young, 2006). Commonly used catchment characteristics include a range of different variables such as catchment area, soil-type, drainage path length, altitude and aridity (McIntyre et al., 2005; Young, 2006; Ye et al., 2014).

The efficacy of regionalization relationships is often compromised by inter-dependence between the model parameters themselves. This is because the inter-dependence increases the variance in the model parameter estimates. Furthermore it is difficult to develop a statistical relationship with catchment characteristics that maintains the complexity of the inter-dependence (McIntyre et al., 2005). These issues become worse with increasing number of model parameters. Various strategies have been proposed to manage these issues, including regionalization schemes that encompass the parameter inter-dependencies, or remove them, or screening of candidate rainfall-runoff model structures that achieve an acceptable balance between simplicity and capability (Lee et al., 2005).

Most rainfall-runoff models comprise at least two components (Wagener et al., 2001): (1) a soil moisture accounting process, used to calculate actual evapotranspiration and runoff generation; (2) a routing function, which transforms the runoff data into an estimate of flow rate at the catchment outlet. The soil moisture accounting process typically requires at least two model parameters, one for

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* Corresponding author.
E-mail address: s.a.mathias@durham.ac.uk (S.A. Mathias).

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the capacity of the soil moisture store and another to help describe how actual evapotranspiration and runoff generation change as the catchment becomes progressively dryer (Lee et al., 2005). The routing function is commonly based on a network of stores each with a defined relationship between storage and outflow. Most commonly, at least when using daily rainfall-runoff data, the network comprises of two linear stores in parallel, conceptually representing the storm-flow and base-flow responses. This routing model requires three parameters: two residence times (one for each store) and a weighting factor defining the proportion of the runoff generation going to each store (Lee et al., 2005).

The perceived requirement of two residence times is often attributed to the existence of two modes of behavior: base-flow and storm-flow (Shaw et al., 2010; Beven, 2012). Base-flow is considered to be due to a slower acting set of hydrological pathways associated with subsurface flow. Conversely, storm-flow is considered to be a faster component associated with flow through surface channel networks. From a calibration perspective, base-flow is required to satisfy the low flow rates observed during dry periods whereas storm-flow is required to simulate the high flow episodes that follow specific storm events.

Although conceptually simple, using a soil moisture store combined with two linear routing stores has had mixed success in terms of well-identified regionalization relationships. Challenges that have been encountered include the co-dependence of the weighting factor and the storm-flow residence time, and the high uncertainty in the base-flow residence time (Lee, 2006). Only using one non-linear routing store, with two parameters rather than three, is one approach to seeking a more identifiable regionalization relationship. A number of studies have demonstrated that a single non-linear store can match the performance of more complex routing functions in some types of gauged catchment (McIntyre, 2013).

The most commonly used non-linear routing store equation (e.g. Wittenberg, 1999; McIntyre et al., 2011; McIntyre, 2013; Ye et al., 2014) takes the form of a well established concept, that river flow can be approximated as a power law of the volume of water stored in the catchment, i.e., (Horton, 1945; Brutsaert and Nieber, 1977)

$$q = aV^b \tag{1}$$

where q [L T^{-1}] is the river flow rate per unit area of catchment, V [L] is the volume of water stored per unit area of catchment and a [L^{1-b} T^{-1}] and b [–] are empirical coefficients.

Considering overland sheet flow, Horton (1945) shows that under laminar conditions (using Poiseuille's law), $b = 3$ and under turbulent conditions (using the Manning formula), $b = 5/3$. Alternatively, assuming that flow occurs through an unconfined aquifer, Brutsaert and Nieber (1977) show (using Darcy's law in conjunction with the Dupuit assumption, i.e., the Boussinesq equation) that $b = 2$.

The power law equation is commonly substituted into a mass conservation statement for the catchment. During recession periods (i.e., periods of negligible runoff generation), application of the chain-rule leads to a direct relationship between flow rate and the rate in change of flow rate

$$\frac{dq}{dt} = -\alpha q^{\beta} \tag{2}$$

where t [T] is time and α [L$^{1-\beta}$ T$^{\beta-2}$] and β [–] can be found from:

$$\alpha = a^{1/b} b \quad \text{and} \quad \beta = \frac{2b-1}{b} \tag{3}$$

and the following inverse relationships apply:

$$a = [\alpha(2-\beta)]^{1/(2-\beta)} \quad \text{and} \quad b = \frac{1}{2-\beta} \tag{4}$$

Note, from Eq. (3), it can be seen that $\lim_{b\to\infty}\beta = 2$.

For a given set of discrete flow measurements, q_n [L T^{-1}], the coefficients α and β can be obtained by linear regression of an approximate form of Eq. (2) (Brutsaert and Nieber, 1977):

$$\ln\left(\frac{q_{n-1} - q_n}{t_n - t_{n-1}}\right) = \ln\alpha + \beta\ln\left(\frac{q_n + q_{n-1}}{2}\right) \tag{5}$$

The potential for reducing uncertainty in regionalization relationships makes the single non-linear store model a potentially attractive replacement for more complex routing models. However, there have been few empirical studies to explore how catchment characteristics control non-linearity in flow routing and whether the strength of these relationships permits a regional model to be proposed.

Ali et al. (2014) constructed a physically based hill-slope model to explore relationships between α, β and their physically based model parameters, by fitting Eq. (5) to results from multiple realizations of the physically based model. Step-wise linear regression analysis suggested that α and β were most sensitive to topographic slope, surface hydraulic conductivity, and the vertical exponential rate of decay for saturated hydraulic conductivity.

Ye et al. (2014) fitted Eq. (5) to recessions from daily flow data series from 50 river catchments from the eastern United States. They then provided a sensitivity analysis for α and β with respect to a range of different catchment characteristics including aridity index, drainage area, topographic slope, drainage density, soil water storage capacity, mean and standard deviation of surface saturated hydraulic conductivity and vertical exponential rate of decay for saturated hydraulic conductivity. It was found that α showed a strong sensitivity to a number of catchment characteristics including soil water storage capacity and surface saturated hydraulic conductivity. In contrast, β showed sensitivity only to aridity index and the rate of decay for saturated hydraulic conductivity.

The developed regression relationships of Ye et al. (2014) suggest that the non-linearity of catchment recession response increases with decreasing aridity and increasing soil hydraulic conductivity decline with depth. The highlighted importance of aridity here is cited as representing an important inconsistency with the results obtained by studying the hill-slope model in Ali et al. (2014) study.

A difficulty with the approach used by Ye et al. (2014) study is that the application of Eq. (5) requires that much of the data set is ignored so as to ensure that all flow data used can be solely attributed to recession. Furthermore, there are many different methods available within the literature for excluding flow data in this way (e.g. Brutsaert and Nieber, 1977; Rupp and Selker, 2006; Kirchner, 2009), and these can lead to variations in α and β on the order of those expected by varying catchment characteristics (Stoelzle et al., 2013).

In this article 120 UK catchments, previously studied by McIntyre et al. (2005) and Young (2006), are revisited to further explore the role of catchment characteristics on non-linearity in rainfall-runoff response. This study builds on the existing work of Ye et al. (2014) by considering a broader range of catchment characteristics, commonly associated with the UK flood estimation handbook (Robson and Reed, 1999). A particular question arising with the UK data set is whether the lumping of storm- and base-flow responses into one conceptual store, as opposed to the conventional parallel stores used in the UK, can lead to meaningful relationships between parameters α and β and the CCs. While intuitively the answer is 'no', the lack of nationally available CCs describing hydrogeology means that regionalization of a base-flow residence time parameter is problematic anyway (Lee et al., 2005; Lee, 2006), and whether the recession and non-recession response can be modeled more holistically and parsimoniously