Journal of Hydrology 540 (2016) 641–658

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Research papers

A mathematical model of pan evaporation under steady state conditions

Wee Ho Lim [a,c,*], Michael L. Roderick [a,b,c], Graham D. Farquhar [a,c]

[a] Research School of Biology, The Australian National University, Canberra, ACT 0200, Australia
[b] Research School of Earth Sciences, The Australian National University, Canberra, ACT 0200, Australia
[c] Australian Research Council Centre of Excellence for Climate System Science, Canberra, Australia

ARTICLE INFO

Article history:
Received 15 March 2016
Received in revised form 21 June 2016
Accepted 22 June 2016
Available online 24 June 2016
This manuscript was handled by Tim McVicar EiC, Editor-in-Chief, with the assistance of Dawen Yang and Matt McCabe, Associate Editors

Keywords:
Pan evaporation
Aerodynamic function
Net irradiance
Short-wave irradiance
Long-wave irradiance

ABSTRACT

In the context of changing climate, global pan evaporation records have shown a spatially-averaged trend of ∼−2 to ∼−3 mm a^{-2} over the past 30–50 years. This global phenomenon has motivated the development of the "PenPan" model (Rotstayn et al., 2006). However, the original PenPan model has yet to receive an independent experimental evaluation. Hence, we constructed an instrumented US Class A pan at Canberra Airport (Australia) and monitored it over a three-year period (2007–2010) to uncover the physics of pan evaporation under non-steady state conditions.

The experimental investigations of pan evaporation enabled theoretical formulation and parameterisation of the aerodynamic function considering the wind, properties of air and (with or without) the bird guard effect. The energy balance investigation allowed for detailed formulation of the short- and long-wave radiation associated with the albedos and the emissivities of the pan water surface and the pan wall. Here, we synthesise and generalise those earlier works to develop a new model called the "PenPan-V2" model for application under steady state conditions (i.e., uses a monthly time step). Two versions (PenPan-V2C and PenPan-V2S) are tested using pan evaporation data available across the Australian continent. Both versions outperformed the original PenPan model with better representation of both the evaporation rate and the underlying physics of a US Class A pan. The results show the improved solar geometry related calculations (e.g., albedo, area) for the pan system led to a clear improvement in representing the seasonal cycle of pan evaporation. For general applications, the PenPan-V2S is simpler and suited for applications including an evaluation of long-term trends in pan evaporation.

1. Introduction

Traditionally, pan evaporation measurements have been used for gauging the evaporative demand of the atmosphere over terrestrial surfaces for practical applications in water resources planning and management, particularly crop irrigation scheduling (Allen et al., 1998) and open water evaporation estimation (Kohler et al., 1955). Pan evaporation networks have been established and maintained across many regions worldwide because of their simplicity and cost effectiveness (Stanhill, 2002). The decline in pan evaporation across many regions worldwide over the past several decades (Peterson et al., 1995) is one of the most interesting of the observed trends yet identified that are associated in some way with climate change. It is of special interest because of its persistent decline across both energy- and water-limited regions (Roderick et al., 2009a; McVicar et al., 2012) and is concurrent with the trend of rising global averaged air temperature. This global phenomenon has attracted broad interest among the scientific community on the cause(s) and its implication(s) for the global hydrologic cycle (e.g., Peterson et al., 1995; Brutsaert and Parlange, 1998; Roderick and Farquhar, 2002; Ohmura and Wild, 2002). A reduction in solar irradiance (Roderick and Farquhar, 2002) and/or wind speed (Roderick et al., 2007) have been identified as the main reasons for those declines (Roderick et al., 2009b; McVicar et al., 2012).

Motivated by the prospect of utilising the long-term pan evaporation records as a scientific database for evaluating the outputs of General Circulation Models (GCMs), Rotstayn et al. (2006) combined the works of Linacre (1994) and Thom et al. (1981) to develop a steady state pan evaporation model for a US Class A pan called the "PenPan" model (Note: The two capital P's in "PenPan" distinguish it from Linacre's earlier contribution called "Penpan"). Using meteorological data (i.e., short-wave and

* Corresponding author at: Environmental Change Institute, Oxford University Centre for the Environment, School of Geography and the Environment, University of Oxford, Oxford OX1 3QY, United Kingdom.

E-mail address: weeho.lim@eci.ox.ac.uk (W.H. Lim).

http://dx.doi.org/10.1016/j.jhydrol.2016.06.048
0022-1694/

long-wave radiation, air temperature, air vapour pressure, wind speed) as inputs, the original PenPan model was tested against observations in Australia (Roderick et al., 2007). However, the physical basis of the original PenPan model is limited by: (i) empirical "wind function" with limited physical understanding, (ii) single pan albedo assumption, i.e., ignoring the fact that the albedos for the pan water surface and the pan wall are different, (iii) ignoring the long-wave radiation exchange of the pan wall with its surroundings, i.e., the sky and the ground surface, and (iv) the assumption that the pan is full of water whereas a typical pan is partially filled (e.g., water level ~0.2 m for a standard US Class A pan (Allen et al., 1998)). Therefore, an independent study is needed to assess and improve the underlying physics represented by the original PenPan model.

We constructed a highly instrumented US Class A pan that replicated an operational pan at Canberra Airport in Australia to study the physics of pan evaporation under non-steady state conditions. In earlier studies, we performed rigorous investigations of the aerodynamics (Lim et al., 2012) and energy balance (Lim et al., 2013) at our experimental pan under non-steady state conditions. Here, we extend those studies through synthesising and upscaling short-term process-level understanding of those formulations into a model suitable for use over time periods long enough for a steady state approximation to be valid (i.e., monthly). We describe the key aspects of improvement of the new formulation over the original PenPan model in Table 1. In Section 2, we synthesise the aerodynamic function of Lim et al. (2012) and radiative formulations of Lim et al. (2013) into a Penman-type combination equation. In particular, we generalise the aerodynamic function; and present a new formulation of the radiative exchanges that accounts for different fractional interception by the water surface relative to the external walls of the pan. The new formulation is called the "PenPan-V2" model. In Section 3, we describe the pan evaporation data used for model evaluation. In Section 4, we evaluate two versions of model developed here (i.e., PenPan-V2C for "Complete"; PenPan-V2S for "Simplified") against observations and demonstrate their advantages over the original PenPan model. In Section 5, we discuss and summarise the outcomes of the research.

2. Model formulation

The central challenge in estimating wet surface evaporation is the lack of measurements. Whilst standard micrometeorological measurements (e.g., radiation, air temperature, air vapour pressure, atmospheric pressure) might be available sometimes, the temperature (or vapour pressure) of the evaporating surface is usually unavailable. To resolve this issue, the wet surface evaporation is typically formulated using an energy balance method where the surface temperature is eliminated from the underlying equations (Penman, 1948) (see simple example in Appendix A). Following that, we use the energy-balance approach to express the general equation for pan evaporation E_{pan} [m s^{-1}] under non-steady state conditions as

$$E_{pan} = \frac{(R_n - \Delta Q)}{\lambda \rho_w} - \frac{H_n}{\lambda \rho_w} \tag{1}$$

where λ [J kg^{-1}] is the latent heat of vaporisation of liquid water (~2.45 MJ kg^{-1}), ρ_w [kg m^{-3}] is the density of liquid water (≈1000 kg m^{-3}), R_n [W m^{-2}] is the net irradiance of the pan, ΔQ [W m^{-2}] is the increase in the heat storage of the bulk water and H_n [W m^{-2}] is the net sensible heat loss of the pan. An equation to calculate λ as a function of air temperature is given in Appendix B.

Following Appendix A, we expand the sensible heat term (i.e., $-\frac{H_n}{\lambda \rho_w}$) and formulate the pan evaporation E_{pan} (Eq. (1)) under steady state conditions (i.e., $\Delta Q \approx 0$). We reorganise E_{pan} into the radiative component ($E_{pan,R}$) and the aerodynamic component ($E_{pan,A}$), i.e.,

$$\begin{aligned} E_{pan} &\approx \frac{R_n}{\lambda \rho_w} + \frac{\beta\gamma}{s+\beta\gamma}\left[f_v(e_s(T_a) - e_a(T_a)) - \frac{R_n}{\lambda \rho_w}\right] \\ &= \underbrace{\frac{s}{s+\beta\gamma}\frac{R_n}{\lambda \rho_w}}_{E_{pan,R}} + \underbrace{\frac{\beta\gamma}{s+\beta\gamma}[f_v(e_s(T_a) - e_a(T_a))]}_{E_{pan,A}} \end{aligned} \tag{2}$$

where β is the ratio of heat to mass transfer coefficients of the pan, γ [Pa K^{-1}] is the psychrometric constant (~67 Pa K^{-1}), s [Pa K^{-1}] is the slope of the saturation vapour pressure versus temperature curve at air temperature (T_a, [K]), f_v [m s^{-1} Pa^{-1}] is the aerodynamic function, $e_s(T_a)$ [Pa] is the saturated vapour pressure at the same height at which air temperature is measured, $e_a(T_a)$ [Pa] is the air vapour pressure at that same height, $E_{pan,R}$ [W m^{-2}] is the radiative component of pan evaporation and $E_{pan,A}$ [W m^{-2}] is the aerodynamic component of pan evaporation. The equations to calculate s and γ are given in Appendix B. For a US Class A pan, β was previously deduced as "the ratio of the total effective surface area of the pan participating in sensible heat exchange to its water surface area" and was revised from a value as large as 2.5 (Kohler et al., 1955) downward to 2.1 (Thom et al., 1981). Through the open water surface example in Appendix A (where the surface areas for both heat and mass transfer are the same), we note that the calculation of β needs to account for the fact that the thermal diffusivity of air (D_h) and the diffusion coefficient for water vapour in air (D_v) are different, i.e., the ratio $\frac{D_h}{D_v} \approx 0.87$. Using the value (2.1) from Thom et al. (1981), we estimate $\beta = 2.1 \times 0.87 \approx 1.8$ and apply this value throughout this paper.

Table 1
Key aspects of improvement in the PenPan-V2 models over the PenPan model.

	PenPan model (Rotstayn et al., 2006)	PenPan-V2 models (Current study)
1. Aerodynamic function	Empirical "wind function"	Boundary layer theory that is parameterised using experimental results
2. Net irradiance		
(a) Short-wave	One constant albedo and area subjected to beam and diffuse irradiance for the entire pan	Different albedos for the pan water surface and the pan wall. Albedo and "effective" area subjected to direct irradiance vary on a monthly basis; constant albedo and area subjected to diffuse irradiance
(b) Long-wave	Long-wave radiation exchange occurs at the pan water surface only. Assumed black body emission for the entire pan	Long-wave radiation exchange occurs at the pan water surface and the pan wall. The pan water surface and the pan wall have different (hemispherical) emissivities
3. Boundary conditions	Pan is full of water	Pan is partially filled with modifications to irradiance. Details in Appendix C
4. Building blocks	Thom et al. (1981) and Linacre (1994)	Lim et al. (2012, 2013)