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Testing the generalized complementary relationship of evaporation with continental-scale long-term water-balance data

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ABSTRACT

The original and revised versions of the generalized complementary relationship (GCR) of evaporation (*ET*) were tested with six-digit Hydrologic Unit Code (HUC6) level long-term (1981–2010) water-balance data (sample size of 334). The two versions of the GCR were calibrated with Parameter-Elevation Regressions on Independent Slopes Model (PRISM) mean annual precipitation (*P*) data and validated against water-balance ET (ET_{wb}) as the difference of mean annual HUC6-averaged *P* and United States Geological Survey HUC6 runoff (*Q*) rates. The original GCR overestimates *P* in about 18% of the PRISM grid points covering the contiguous United States in contrast with 12% of the revised version. With HUC6-averaged data the original version has a bias of -25 mm yr^{-1} vs the revised version's -17 mm yr^{-1}, and it tends to more significantly underestimate ET_{wb} at high values than the revised one (slope of the best fit line is 0.78 vs 0.91). At the same time it slightly outperforms the revised version in terms of the linear correlation coefficient (0.94 vs 0.93) and the root-mean-square error (90 vs 92 mm yr^{-1}).

1. Introduction

The Complementary Relationship (CR) of evaporation (Bouchet, 1963) is a physically based approach that yields actual evapotranspiration rates (*ET*) without detailed knowledge of the land-surface properties: terrain, land-cover and soil type, as well as soil-moisture status. This makes the CR undeniably appealing. The CR implicitly accounts for the complex interplay between the land, vegetation, and atmospheric components by relating the typically moisture-limited actual *ET* to two energy-limited evaporation rates of fully wet surfaces differing mostly in horizontal extent: one from a small wet patch or a shallow pond or even an evaporation pan (E_p in mm d^{-1}) under the prevailing atmospheric conditions, the other, from a wet landscape (E_w in mm d^{-1}), driven predominantly by the available energy (R_n) at the surface (Priestley and Taylor, 1972)

$$E_w = \alpha \frac{\Delta(T_w)}{\Delta(T_w) + \gamma} R_n. \tag{1}$$

Here R_n is specified in water depth per unit time (mm d^{-1}), Δ (hPa K^{-1}) is the slope of the saturation vapor pressure curve at the wet environment air temperature, T_w (K), and γ [= $c_p p/(0.622L)$ in hPa K^{-1}] is the psychrometric constant, where c_p (J kg^{-1} K^{-1}) is the specific heat of air at constant pressure, p (hPa), and L (J kg^{-1}) is latent heat of vaporization for water. The dimensionless coefficient α (>1) is generally accepted to express the evaporation-enhancing effect of large-scale entrainment of drier free-tropospheric air resulting from the growing daytime convective boundary layer (Brutsaert, 1982; de Bruin, 1983; Culf, 1994; Lhomme, 1997; Heerwaarden et al., 2009). T_w can be estimated from the prevailing air temperature, T_a (K) via an implicit formula based on the Bowen ratio (B_o) written for a small wet surface with daily sums of the fluxes (Szilagyi and Jozsa, 2008) as

$$B_o \approx \frac{R_n - E_p}{E_p} \approx \gamma \frac{T_{ws} - T_a}{e*(T_{ws}) - e(T_a)} \tag{2}$$

making use of the assumption that R_n, T_a and the actual vapor pressure measurements, e (hPa), (e^* its saturated value) are also valid over the wet surface due to the small extent of the latter. T_{ws} (K) is the estimated air temperature at the wet surface, which has been shown to be independent of the horizontal extent (Szilagyi and Schepers, 2014) of the wet patch. Since the equilibrium air

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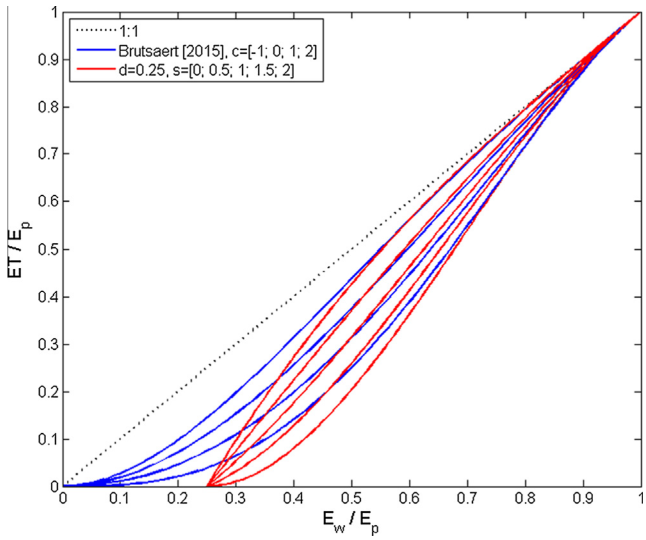

Fig. 1. Theory-predicted E_p-scaled evaporation rates for selected values of the parameters within the physically meaningful ranges of $-1 \leqslant c \leqslant 2$ for (8) and $0 \leqslant d < 1$, $0 \leqslant s \leqslant (2d+1)/(1-d)$ for (9) where thus $ET/E_p \leqslant E_w/E_p$ and ET/E_p monotonically increases with E_w/E_p. The two-parameter curve is valid for $E_w/E_p \geqslant d$ only. Parameter values of c decrease, while those of s increase from bottom to top of the corresponding series of curves.

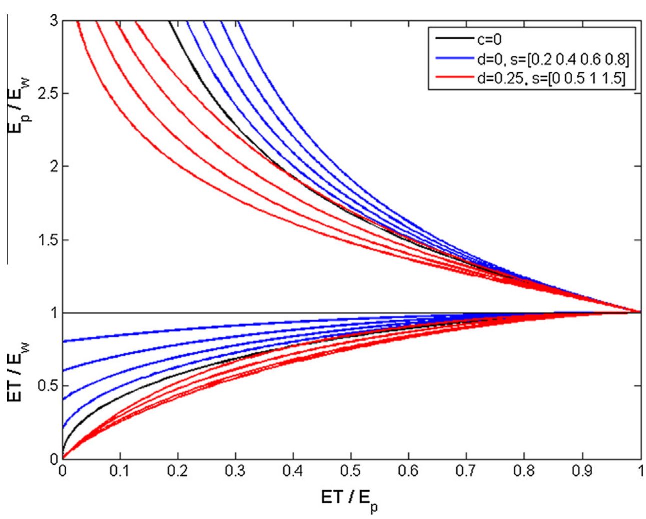

Fig. 2. Theory-predicted E_w-scaled actual and apparent potential evaporation rates for selected values of the parameters as a function of the moisture index, ET/E_p. The values of s increase from bottom to top of the corresponding series of curves.

temperature profile over an extended wet surface has a mild gradient with height above the ground (Szilagyi and Jozsa, 2009), the T_{ws} value estimated from (2) can be taken for T_w in (1) as long as $T_{ws} < T_a$, otherwise T_w can be replaced by T_a (Huntington et al., 2011; McMahon et al., 2013a, 2013b; Szilagyi, 2014a).

In lieu of direct measurements, the so-called apparent potential (Brutsaert, 2005) evaporation rate of the small wet surface can be defined by the Penman (1948) equation

$$E_p = \frac{\Delta(T_a)}{\Delta(T_a)+\gamma}R_n + \frac{\gamma}{\Delta(T_a)+\gamma}f_u[e*(T_a) - e(T_a)] \tag{3}$$

where f_u (mm d^{-1} hPa^{-1}) is an empirical wind function. With the proper choice of f_u, (3) can estimate E_p rates of a small pond, a wet patch or a sunken evaporation pan, i.e.,

$$f_u = 0.26(1 + 0.54u_2) \tag{4}$$

where u_2 is the wind speed (m s^{-1}) at 2 m above the ground (Brutsaert, 1982). As an alternative, monthly E_p rates of class-A evaporation pans can, however, be better estimated by a more sensitive f_u (Szilagyi and Jozsa, 2008)

$$f_u = 0.49(1 + 0.35u_2). \tag{5}$$

The difference in the two energy-limited evaporation rates, i.e., E_w and E_p, is due mainly to edge-effects: the horizontal energy transport [expressed by the second term of (3)] which is significant over the small, freely evaporating wet surface dissipates with the extent of the wet area. The more significant this edge effect, the more arid the environment has become, which forms the basis of the CR, typically written as

$$ET = E_w - (E_P - E_w)/b \tag{6}$$

where b^{-1} (–) (or b itself) is a proportionality coefficient (Kahler and Brutsaert, 2006; Szilagyi, 2007; Zuo et al., 2015). Kahler and Brutsaert (2006) assumed it to be a constant, i.e., $b = 4.5$, when employing class-A evaporation pans for obtaining E_p. The CR with the so-called Rome wind-function of (4) is typically found to be symmetric, i.e., $b = 1$ (Ramirez et al., 2005). Recently Szilagyi (2015) related b^{-1} to the relative humidity of the air via a modified logistic curve, using (5). Note that the three evaporation rates collapse into one under energy-limited conditions, i.e., $ET = E_w = E_p$.

An exciting new generalization of the CR has been developed recently by Brutsaert (2015) who originally spearheaded the employment of the complementary relationship for practical applications (Brutsaert and Stricker, 1979). The objectives of this study are (i) to introduce a revision to this newly generalized CR; and (ii) to compare the performance of the two versions via calibration with precipitation and validation against long-term (1981–2010) water-balance derived mean annual evaporation rates across the conterminous United States.

2. Theoretical background: The generalized complementary relationship (GCR)

Brutsaert (2015) reformulated the CR by setting physical constraints for the relationship between E_p-scaled actual ($y = ET/E_p$) and wet environment evaporations ($x = E_w/E_p$) in the form of boundary conditions (BC), inspired by the work of Han et al. (2012). The four BCs he employed for an assumed polynomial relationship between y and x are

$$\text{(i) } y = 1 \text{ at } x = 1; \text{(ii) } y = 0 \text{ at } x = 0; \text{(iii) } dy/dx = 1 \text{ at } x = 1; \text{(iv) } dy/dx = 0 \text{ at } x = 0. \tag{7}$$

BC (i) results from $ET = E_w = E_p$ under energy-limited conditions. BC (iii) is consistent with E_p values that are more sensitive to changes in moisture availability than ET itself, as x approaches unity. Brutsaert (2015) also introduced an additional parameter c (–) into the BC-derived polynomial relationship for added flexibility. The resulting relationship has become

$$y = (2-c)x^2 - (1-2c)x^3 - cx^4 \tag{8}$$

3. Methods

3.1. Revision of the GCR

(8) must satisfy two additional constraints (a) y must be monotonically increasing with respect to x; and (b) $y \leqslant x$ for $0 \leqslant x \leqslant 1$. These latter two conditions restrain the admissible range of c to $-1 \leqslant c \leqslant 2$, which is important to define for practical applications