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Research papers

A general analytical model for pumping tests in radial finite two-zone confined aquifers with Robin-type outer boundary

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ABSTRACT

This study develops a general analytical model for describing transient drawdown distribution induced by pumping at a finite-radius well in a radial two-zone confined aquifer of finite areal extent with Robin-type condition at both inner and outer boundaries. This model is also applicable to heat conduction problems for a composite hollow cylinder on the basis of the analogy between heat flow and groundwater flow. The time-domain solution of the model is derived by the methods of Laplace transform, Bromwich integral, and residue theorem. This new solution can reduce to the solution for constant-head test (CHT) or constant-rate test (CRT) problem by specifying appropriate coefficients at the Robin inner boundary condition. The solution describing the flow rate across the wellbore due to CHT is further developed by applying Darcy's law to the new solution. In addition, steady-state solutions for both CHT and CRT are also developed based on the approximation for Bessel functions with very small argument values. Many existing solutions for transient flow in homogeneous or two-zone finite aquifers with Dirichlet or no-flow condition at the outer boundary are shown to be special cases of the present solution. Furthermore, the sensitivity analysis is also performed to investigate the behaviors of the wellbore flow due to CHT and the aquifer drawdown induced by CRT in response to the change in each of aquifer parameters.

1. Introduction

Aquifer tests such as constant-head test (CHT) and constant-rate test (CRT) are commonly performed for the determination of aquifer parameters (i.e., transmissivity and storage coefficient) in engineering applications. The former is carried out by maintaining a constant water level in a well and measuring flow rate across the wellbore. The latter needs to maintain a constant-flux rate at the pumping well and measures the temporal change of the drawdown at the observation well. The aquifer parameters can then be determined by the least-squares or graphical approaches with the measured data.

A variety of studies describing the transient drawdown behavior of CHT in confined homogeneous aquifers under various well and aquifer configurations has been reported in the past. Muskat (1946) presented two mathematical models for describing well pumping at a uniform rate and a constant pressure in a closed gas reservoir with no-flow condition at the outer boundary. In petroleum engineering area, van Everdingen and Hurst (1949) developed analytical solutions for flow problems with considering both constant-pressure and constant-rate cases in reservoirs with zero pressure drop at the remote boundary for both finite and infinite boundary systems. Jacob and Lohman (1952), based on the solution of Smith (1937) for heat conduction problem, showed a formula describing wellbore flow rate induced by constant drawdown in a confined infinite aquifer by specifying an initial head at the outer boundary. Later, Carslaw and Jaeger (1959) presented mathematical models for both constant temperature and constant heat flux as inner boundary condition (BC) for heat flow problems with zero initial temperature over the entire infinite medium. Their solutions can be adopted as head solutions for CHT and CRT in a confined aquifer on the basis of analogy between groundwater flow and heat conduction. In addition, they also provided temperature distribution solution for a hollow cylinder with the Robin conditions at both the inner and outer boundaries (Carslaw and Jaeger, 1959, p. 332). Mishra and Guyonnet (1992) adopted the Boltzmann transformation technique to develop approximate solutions for drawdown and wellbore flow-rate solutions for homogeneous aquifers. Markle et al. (1995) developed a

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Nomenclature

$J_0(u), Y_0(u)$ Bessel function of the first and second kinds of order zero
$J_1(u), Y_1(u)$ Bessel function of the first and second kinds of order one
$I_0(u), K_0(u)$ modified Bessel function of the first and second kinds of order zero
$I_1(u), K_1(u)$ modified bessel function of the first and second kinds of order one
O output parameter of the aquifer
P input parameter of the aquifer
$Q(r_w)$ flow rate into or out wellbore
$\overline{Q}_D$ dimensionless flow rate in Laplace domain
$\overline{Q}(r_w)$ flow rate in Laplace domain
R radial distance from the center of well to remote boundary
S storage coefficient
T transmissivity
X normalized sensitivity
B thickness of aquifer
b' thickness of streambed
g s_w for CHT and $Q/(4\pi T_2)$ for CRT
k coefficients of inner and outer BCs
k' permeability of streambed
p Laplace variable
q $= i\sqrt{pS/T}$
q_{1D} $= \sqrt{\frac{\kappa}{\sigma}p}$
q_{2D} $= \sqrt{p}$
r radial distance from central line of the test well
r_1 thickness of the skin zone
r_w well radius
s drawdown
s_w. constant drawdown at pumping well
$\bar{s}$ drawdown in Laplace domain
t time from the begin of pumping
ΔP_k $= 10^{-3}P_k$ increment of the k-th parameter P_k
Δ $= J_0(\lambda_n)[\lambda_n\alpha_2\beta_1\mu_1 + \lambda_n\beta_2\beta_1\mu_2 + \alpha_2\alpha_1\mu_1' + \beta_2\alpha_1\mu_2'] + J_1(\lambda_n)[\lambda_n\beta_1\alpha_2\mu_1' + \lambda_n\beta_1\beta_2\mu_2' - \alpha_2\alpha_1\mu_1 - \beta_2\alpha_1\mu_2] + Y_0(\lambda_n)[\lambda_n\alpha_2\beta_1\mu_5 + \lambda_n\beta_2\beta_1\mu_6 + \alpha_2\alpha_1\mu_5' + \beta_2\alpha_1\mu_6'] + Y_1(\lambda_n)[\lambda_n\beta_1\alpha_2\mu_5' + \lambda_n\beta_1\beta_2\mu_6' - \alpha_2\alpha_1\mu_5 - \beta_2\alpha_1\mu_6]$ in Eq. (A49)
Δ_H $= -J_1(\lambda_n)(\kappa'\mu_1 + \mu_2) - Y_1(\lambda_n)(\kappa'\mu_5 + \mu_6) + J_0(\lambda_n)(\kappa'\mu_1' + \mu_2') + Y_0(\lambda_n)(\kappa'\mu_5' + \mu_6')$ in Eq. (37)
Δ_R $= \lambda_n[J_0(\lambda_n)(\kappa'\mu_1 + \mu_2) + Y_0(\lambda_n)(\kappa'\mu_5 + \mu_6) + J_1(\lambda_n)(\kappa'\mu_1' + \mu_2') + Y_1(\lambda_n)(\kappa'\mu_5' + \mu_6')]$ in Eq. (51)
H $= (\gamma_1\Phi_1 + \gamma_2\Phi_2)K_0(q_{1D}\rho) + (\gamma_1\Phi_3 + \gamma_2\Phi_4)I_0(q_{1D}\rho)$ in Eq. (B3)
M_1 $= \alpha_2\mu_1 + \beta_2\mu_2$ in Eq. (A25)
M_2 $= \alpha_1\mu_3 + \beta_1\mu_4$ in Eq. (A26)
M_3 $= \alpha_2\mu_5 + \beta_2\mu_6$ in Eq. (A27)
M_4 $= \alpha_2\mu_7 + \beta_2\mu_8$ in Eq. (A28)
N_1 $= \alpha_2 v_1 + \beta_2 v_2$ in Eq. (A29)
N_2 $= \alpha_1 v_3 + \beta_1 v_4$ in Eq. (A30)
N_3 $= \alpha_2 v_5 + \beta_2 v_6$ in Eq. (A31)
N_4 $= \alpha_2 v_7 + \beta_2 v_8$ in Eq. (A32)
Φ_1 $= \alpha_2\phi_1 + \beta_2\phi_2$ in Eq. (A1)
Φ_2 $= \alpha_1\phi_3 + \beta_1\phi_4$ in Eq. (A2)
Φ_3 $= \alpha_2\phi_5 + \beta_2\phi_6$ in Eq. (A3)
Φ_4 $= \alpha_2\phi_7 + \beta_2\phi_8$ in Eq. (A4)
Ω_1 $= \alpha_2\varphi_1 + \beta_2\varphi_2$ in Eq. (A5)
Ω_2 $= \alpha_1\varphi_3 + \beta_1\varphi_4$ in Eq. (A6)
Ω_3 $= \alpha_2\varphi_5 + \beta_2\varphi_6$ in Eq. (A7)
Ω_4 $= \alpha_2\varphi_7 + \beta_2\varphi_8$ in Eq. (A8)
α_1 constant value
α_2 constant value
β_1 constant value
β_2 constant value
γ_1 constant value
γ_2 constant value
κ $= T_2/T_1$ in Eq. (8)
κ' $= k'br_w/T_2b'$
λ_n the roots of $\psi = 0$
μ_1 $= q_{2D}\kappa J_1(q_{2D}\rho_1)Y_0(q_{2D}\rho_R)Y_0(q_{1D}\rho_1) - q_{2D}\kappa J_0(q_{2D}\rho_R)Y_0(q_{1D}\rho_1)Y_1(q_{2D}\rho_1) + q_{1D}[J_0(q_{2D}\rho_R)Y_0(q_{2D}\rho_1) - J_0(q_{2D}\rho_1)Y_0(q_{2D}\rho_R)]$ in Eq. (A33)
μ_2 $= q_{2D}^2\kappa J_1(q_{2D}\rho_1)Y_0(q_{1D}\rho_1)Y_0(q_{2D}\rho_R) + q_{1D}q_{2D}Y_1(q_{1D}\rho_1)[J_1(q_{2D}\rho_R)Y_0(q_{2D}\rho_1) - J_0(q_{2D}\rho_1)Y_1(q_{2D}\rho_R)] - q_{2D}^2\kappa J_1(q_{2D}\rho_R)Y_0(q_{1D}\rho_1)Y_1(q_{2D}\rho_1)$ in Eq. (A34)
μ_3 $= -\frac{2q_{1D}\kappa Y_1(q_{1D})}{\pi\rho_1}$ in Eq. (A35)
μ_4 $= -\frac{2\kappa Y_0(q_{1D})}{\pi\rho_1}$ in Eq. (A36)
μ_5 $= q_{1D}J_0(q_{2D}\rho_1)J_1(q_{1D}\rho_1)Y_0(q_{2D}\rho_R) - q_{1D}J_0(q_{2D}\rho_R)J_1(q_{1D}\rho_1)Y_0(q_{2D}\rho_1) + q_{2D}\kappa J_0(q_{1D}\rho_1)[J_0(q_{2D}\rho_R)Y_1(q_{2D}\rho_1) - J_1(q_{2D}\rho_1)Y_0(q_{2D}\rho_R)]$ in Eq. (A37)
μ_6 $= q_{2D}Y_1(q_{2D}\rho_R)\begin{bmatrix} q_{1D}J_0(q_{2D}\rho_1)J_1(q_{1D}\rho_1) \\ -q_{2D}\kappa J_0(q_{1D}\rho_1)J_1(q_{2D}\rho_1) \end{bmatrix} - q_{2D}J_1(q_{2D}\rho_R)\begin{bmatrix} q_{1D}J_0(q_{1D}\rho_1)Y_0(q_{2D}\rho_1) \\ -q_{2D}\kappa J_0(q_{1D}\rho_1)Y_1(q_{2D}\rho_1) \end{bmatrix}$ in Eq. (A38)
μ_8 $= -\frac{2q_{1D}\kappa J_1(q_{1D})}{\pi\rho_1}$ in Eq. (A39)
μ_7 $= \frac{2\kappa J_0(q_{1D})}{\pi\rho_1}$ in Eq. (A40)
v_1 $= \frac{-2Y_0(q_{2D}\rho_R)}{\pi\rho_1}$ in Eq. (A41)
v_2 $= \frac{2q_{2D}Y_1(q_{2D}\rho_R)}{\pi\rho_1}$ in Eq. (A42)
v_3 $= q_{1D}Y_0(q_{2D}\rho_1)[J_0(q_{1D})Y_1(q_{1D}\rho_1) - J_1(q_{1D}\rho_1)Y_0(q_{1D})] + q_{2D}\kappa Y_1(q_{2D}\rho_1)[J_0(q_{1D}\rho_1)Y_0(q_{1D}) - J_0(q_{1D})Y_0(q_{1D}\rho_1)]$ in Eq. (A43)
v_4 $= q_{1D}^2Y_0(q_{2D}\rho_1)[J_1(q_{1D})Y_1(q_{1D}\rho_1) - J_1(q_{1D}\rho_1)Y_0(q_{1D})] + q_{1D}q_{2D}\kappa Y_1(q_{2D}\rho_1)[J_0(q_{1D}\rho_1)Y_1(q_{1D}) - J_1(q_{1D})Y_0(q_{1D}\rho_1)]$ in Eq. (A44)
v_5 $= -\frac{2J_0(q_{2D}\rho_R)}{\pi\rho_1}$ in Eq. (A45)
v_6 $= -\frac{2q_{2D}J_1(q_{2D}\rho_R)}{\pi\rho_1}$ in Eq. (A46)
v_7 $= q_{2D}\kappa J_1(q_{2D}\rho_1)[J_0(q_{1D})Y_0(q_{1D}\rho_1) - J_0(q_{1D}\rho_1)Y_0(q_{1D})] q_{1D}J_0(q_{2D}\rho_1)[J_1(q_{1D}\rho_1)Y_0(q_{1D}) - J_0(q_{1D})Y_1(q_{1D}\rho_1)]$ in Eq. (A47)
v_8 $= q_{1D}^2J_0(q_{2D}\rho_1)J_1(q_{1D}\rho_1)Y_1(q_{1D}) - q_{1D}^2J_0(q_{2D}\rho_1)J_1(q_{1D})Y_1(q_{1D}\rho_1) + q_{1D}q_{2D}\kappa J_1(q_{2D}\rho_1)[J_1(q_{1D})Y_0(q_{1D}\rho_1) - J_0(q_{1D}\rho_1)Y_1(q_{1D})]$ in Eq. (A48)
ρ $= r/r_w$ in Eq. (8)
ρ_1 $= r_1/r_w$ in Eq. (8)
ρ_R $= R/r_w$ in Eq. (8)
σ $= S_2/S_1$ in Eq. (8)
τ $= T_2t/S_2r_w^2$ in Eq. (8)
ϕ_1 $= -q_{1D}I_0(q_{2D}\rho_1)I_1(q_{1D}\rho_1)K_0(q_{2D}\rho_R) + q_{1D}I_0(q_{2D}\rho_R)I_1(q_{1D}\rho_1)K_0(q_{2D}\rho_1) + q_{2D}\kappa I_0(q_{1D}\rho_1)[I_1(q_{2D}\rho_1)K_0(q_{2D}\rho_R) + I_0(q_{2D}\rho_R)K_1(q_{2D}\rho_1)]$ in Eq. (A9)
ϕ_2 $= q_{2D}K_1(q_{2D}\rho_R)[q_{1D}I_0(q_{2D}\rho_1)I_1(q_{1D}\rho_1) - q_{2D}\kappa I_0(q_{1D}\rho_1)I_1(q_{2D}\rho_1)] + q_{2D}I_1(q_{2D}\rho_R)[q_{1D}I_0(q_{1D}\rho_1)K_1(q_{2D}\rho_1) - q_{2D}\kappa I_0(q_{1D}\rho_1)K_1(q_{2D}\rho_1)]$ in Eq. (A10)
ϕ_3 $= -\frac{\kappa I_0(q_{1D})}{\rho_1}$ in Eq. (A11)
ϕ_4 $= -\frac{q_{1D}\kappa I_1(q_{1D})}{\rho_1}$ in Eq. (A12)
ϕ_5 $= -q_{2D}\kappa I_1(q_{2D}\rho_1)K_0(q_{2D}\rho_R)K_0(q_{1D}\rho_1) - q_{2D}\kappa I_0(q_{2D}\rho_R)K_0(q_{1D}\rho_1)K_1(q_{2D}\rho_1) + q_{1D}K_1(q_{1D}\rho_1)[I_0(q_{2D}\rho_R)K_1(q_{2D}\rho_1) - I_0(q_{2D}\rho_1)K_0(q_{2D}\rho_R)]$ in Eq. (A13)