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Time-harmonic response of saturated porous transversely isotropic half-space under surface tractions

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SUMMARY

An analytical investigation is here presented for dynamic analysis of a half-space containing saturated porous transversely isotropic material under surface tractions. The solid displacement-pore fluid pressure formulation of Biot's theory known as $\mathbf{u}-p$ formulation is accepted as the governing equations for the whole half-space, which is the domain of the problem. The free surface of the half-space is considered completely permeable. Two scalar potential functions are for the first time introduced to uncouple the governing system of partial differential equations. The potential functions are introduced in such a way that the governing operators for the potential functions to be physically meaningful. By applying Fourier and Hankel integral transforms, the potential functions are determined by solving two ordinary differential equations. Then, the displacements, stresses and pore fluid pressure are presented as the solutions for the boundary value problem involved in this paper in terms of some line integrals that are evaluated numerically. These responses are derived for general patch load, and presented for horizontal and vertical circular loads, vertical ring load and vertical point load as well as special cases. In addition, the degeneration of the proposed solution to thermoelastodynamics is shown. Selected numerical results for a half-space subjected to uniform horizontal load applied on a circular disc are illustrated.

1. Introduction

Biot's theory has been applied successfully in various poroelastic problems by several authors, both analytically and numerically. The early fundamental solutions of the saturated porous media can be found in (Burridge and Vargas, 1979), where only a single point load acting on the solid phase had been considered. This approach was followed in (Norris, 1985), where the time harmonic Green's functions for a point load in an unbounded fluid-saturated porous media were presented. If Laplace or Fourier integral transform is applied to the governing equations of poroelasticity, then an analogy between the thermoelastic governing equations and Biot's equations for saturated porous media may be made, from which the solution of mono-harmonic dynamic problem of a saturated porous media are determined using the existing solutions for thermoelastic problems (Bonnet, 1987; Manolis and Beskos, 1989).

Zienkiewicz et al. (1980) proposed a simplified formulation of Biot's equations for poroelastodynamic problems in low frequency, where the inertial effect due to the relative acceleration between solid and pore fluid phases is ignored. This is known as $\mathbf{u}-p$ formulation, in which the solid displacement, $\mathbf{u}$, and the pore fluid pressure, p, are primary variables. The $\mathbf{u}-p$ formulation is adequate for analysis of seismic problems; however, it may be less accurate for high frequency ones (Shi et al., 2012; Zienkiewicz et al., 1980). Since $\mathbf{u}-p$ formulation has less unknown variables than $\mathbf{u}-\mathbf{U}$ formulation, where the solid displacement, $\mathbf{u}$, and the fluid displacement, $\mathbf{U}$, are the unknown functions, and also since $\mathbf{u}-p$ formulation is responsive in many engineering fields, where the excitation frequency is not high, it has been more attracted by researchers, especially in computational fields. For instance, one of the fundamental solutions for $\mathbf{u}-p$ formulation has been proposed by Schanz and Struckmeier (2005). In this research, the $\mathbf{u}-p$ formulation of Biot's equations has been transformed to Laplace domain. With the use of the method proposed by Hormander (1963), the problem were reduced to a set of simpler scalar equations from which the fundamental solutions were found. The solution has been used in a convolution quadrature-based boundary element formulation (Schanz and Struckmeier, 2005). Senjuntichai and Rajapakse (1994), with the use of Helmholtz decomposition of a vector field, derived the Green's functions of 2D homogeneous poroelastic half-space in frequency domain. Ganbe and Kurashige (2000) represented the fundamental solutions for an isotropic poroelastic infinite space. Chen et al. (2007) have determined the three dimensional Green's functions of an

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Nomenclature

C_{ijkl}	elasticity constants
E	Young's moduli in the plane of transverse isotropy
E'	Young's moduli in the direction normal to the plane of transverse isotropy
G	shear modulus in the plane normal to the axis of symmetry
G'	shear modulus in planes normal to the plane of transverse isotropy
ν	Poisson's ratio characterizing the lateral strain response in the plane of transverse isotropy to a stress acting parallel to it
ν'	Poisson's ratio characterizing the lateral strain response in the plane of transverse isotropy to a stress acting normal to it
$P(r,\theta)$	surface force component in r-direction
$Q(r,\theta)$	surface force component in θ-direction
$R(r,\theta)$	surface force component in z-direction
a	radius of circular surface load
r	radial coordinate
θ	angular coordinate
z	vertical coordinate
t	time variable
u_r	displacement component in r-direction
u_θ	displacement component in θ-direction
u_z	displacement component in z-direction
$\mathbf{w}$	relative displacement vector of fluid
p	pore fluid pressure
$\lambda_1, \lambda_2, \lambda_3$	radicals appearing in general solutions
ω	angular frequency
ω_0	nondimensional frequency
$\delta(r)$	Dirac-delta function
π_0	patch of surface load
$\sigma_{ij}(i,j=r,\theta,z)$	stress tensor
ζ	variation of fluid content
υ	kinematic viscosity of fluid
d	order of the diameter of the pores
η	dynamic viscosity of the fluid
ρ^s	mass density of matrix
ρ^f	mass density of fluid
ρ	mass density of mixture
k_1	intrinsic permeability in any direction in horizontal plane
k_3	intrinsic permeability in vertical direction
n	porosity
α_1	Biot effective stress coefficient in horizontal plane
α_3	Biot effective stress coefficient in vertical plane
M	Biot modulus
K_s	bulk modulus of solid phase
K_f	bulk modulus of fluid phase
J_m	Bessel function of the first kind and mth order
H_m	mth order of Hankel integral transform
ξ	Hankel's parameter
X_m	$-\bar{P}_m^{m-1} + i\bar{Q}_m^{m-1}$ (P and Q have been defined previously)
Y_m	$-\bar{P}_m^{m+1} - i\bar{Q}_m^{m+1}$ (P and Q have been defined previously)
Z_m	$-\bar{R}_m^m$ (R has been defined previously)
F	Scalar potential function
χ	Scalar potential function
ψ	The body force related potential functions

isotropic saturated half-space under arbitrary buried load. In their research, the Biot's equations have been expressed in cylindrical coordinate system and the general solutions have been obtained by using Fourier expansion and Hankel integral transforms. Green's functions of an infinite transversely isotropic saturated porous media can be found in Kazi-Aoual et al. (1988). The solutions were derived by using the Kupradze's method (Kupradze, 1979), however, the explicit forms of the solutions were not obtained there. Taguchi and Kurashige (2002) have presented the fundamental solutions for a full-space filled by linear elastic permeable fluid-saturated porous transversely isotropic solid, where the Biot's equations have been expressed in Laplace domain and Kupradze's method accompanied with a triple Fourier integral transforms have been utilized to find the solutions.

It is well known that real materials are mostly anisotropic in both mechanical and consequently hydraulic points of view. In this paper, a new analytical approach is proposed to find the elastodynamic solution of a saturated porous half-space, which is considered to be transversely isotropic in view of both mechanical and hydraulic behavior. The $\mathbf{u} - p$ formulation of Biot's equations (Biot, 1955) is considered and two new scalar potential functions are introduced to uncouple the coupled equations of motion and fluid continuity equation. Using these potential functions, the coupled equations of motion and fluid continuity equation are reduced to a sixth order and a second order scalar partial differential equations in any orthogonal coordinate system. Accepting cylindrical coordinate system as a reference in this paper, the Fourier series in terms of circumferential coordinate and Hankel integral transforms in terms of radial coordinate are applied from which two ordinary differential equations are obtained that are solved readily. By introducing solid phase displacement-potential function and pore fluid pressure-potential function relationships in transformed domain, both the solid phase displacements and pore fluid pressure are derived in transformed domain. By using the theorem of inverse Hankel integral transforms, the results can be found in the original physical domain in the forms of some improper line integrals. Since the integrands are complex functions, the integrals have to be evaluated numerically. In this regard, the *Mathematica* software is utilized. The numerical evaluations are presented for the special case of horizontal surface tractions acting on a circular disc, where the validity and accuracy of the solutions presented here may be shown for some simpler cases degenerated from the solution obtained in this paper. Some more numerical results are illustrated for displacements and pore fluid pressure in terms of both radial coordinate and depth, where some comparisons are presented to capture the physical behavior of the problem in hand.

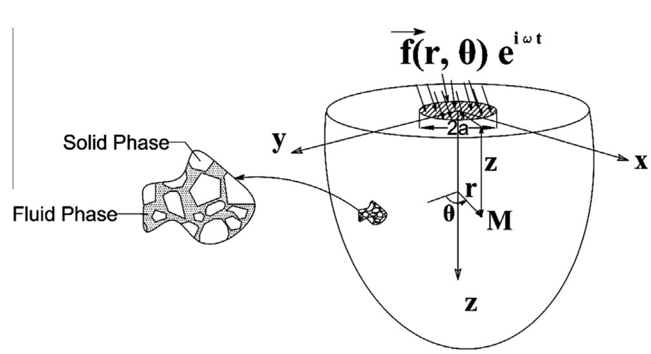

Fig. 1. Saturated porous half space under arbitrary surface traction on a circular disk of radius a.