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Hydraulic dispersion of diurnal reactive constituents in an open channel eutrophic flow

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SUMMARY

Closely related to the solar photocycle, plankton growth in eutrophic waters displays a diurnal variation because of photosynthesis and respiration. Presented in this paper is an analytical study of the diurnal variation of mean concentration of plankton and nutrient in an open channel eutrophic flow initiated by an instantaneous emission. The evolution of the concentration is shown driven by the combination of hydraulic dispersion and diurnal reaction between plankton and nutrient. The analytical solution for longitudinal distribution of concentration is rigorously derived and illustrated, based on the time dependent hydraulic dispersivity. Numerical results are presented and characterized by the reaction rate, yield factor and period for the diurnal reaction and the Péclet number of the flow. For typical applications such as ecological risk assessment and environmental impact assessment, the upper and lower limits of critical length and duration of five typical pollutant concentrations are concretely illustrated for given concentration criterions. Remarkable diurnal variations are revealed up to around one third in the critical length and duration for plankton, and about ten percent for nutrient.

1. Introduction

Eutrophication is a widespread ecological problem impairing rivers, lakes, estuaries and coastal oceans, arises directly from the oversupply of nutrients and pollutants. In the occurrence of water blooms associated with eutrophic waterbodies, plankton growth consumes essential organic and inorganic nutrients (Sunda and Huntsman, 1997; Anderson et al., 2002). Changes in biomass and physicochemical characteristics of aquatic ecosystems in turn generate subordinate concentration variation of nutrients (Koelmans et al., 2001). Accurate analysis and prediction of the concentration of plankton and nutrient plays a key role in ecological risk assessment, environmental impact assessment, and wastewater treatment engineering (Jak et al., 1996; Van den Brink et al., 2009; Wang and Chen, 2016; Wang et al., 2013; Wang and Chen, 2015; Zeng et al., 2014; Wu and Chen, 2014).

In the analysis of concentration transport in open channels, rivers, wetlands, etc., Taylor's dispersion model has been extensively applied (Elder, 1959; Holley et al., 1970; Fischer et al., 1979; Wang et al., 2013; d'Orlye and Reiller, 2012; Wang et al., 2015). The concept of Taylor dispersion refers to the process that dissolved solute spreads longitudinally by a virtual diffusion coefficient that is much larger than the molecular or turbulent eddy diffusivity (Taylor, 1953). After some short initial stage since instantaneous emission, the concentration cloud appears as a longitudinal Gaussian distribution in terms of the depth-average mean concentration, as described by a one-dimensional diffusion-like equation (Chatwin, 1970; Wang and Chen, 2016; Wu et al., 2015). Commonly concerned during this process is to predict the critical length and duration of the influenced region at the large scale where pollutant concentration is beyond some given environmental criterions.

Taylor dispersion model coupled with boundary reaction or bulk degradation has been widely applied as well in the area of environment prevention and conservation to characterize scalar transport involving various biological and physicochemical effects (Wang and Chen, 2015; Barry et al., 2013; Curtis et al., 2010; Chen, 2013; Zeng et al., 2014; Zeng and Chen, 2011). The decay of mean concentration in a wetland flow under the effect of degradation has been theoretically studied (Zeng and Chen, 2011; Chen, 2013; Wang and Chen, 2015). Subsequently, considering the combined action of reversible and irreversible reactions, Zeng et al. (2014) studied the dispersion of bicomponent contaminant in a wetland channel flow. In the existing researches, however, the evolution of scalar concentration shows a monotonous decay after the initial emission, regardless of the effect of plankton uptake and release of nutrient.

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Previous studies have demonstrated that plankton and nutrient concentrations show great fluctuations (Anderson et al., 2008; Nimick et al., 2011; Cornwall et al., 2013). Plankton growth in natural waters is generally synchronous or similar with the diurnal variation of photosynthesis, dominated by sunlight, temperature, pH, and the supply of nutrients (Nimick et al., 2011; Pokrovsky and Shirokova, 2013). The plankton growth and nutrient uptake would generate an increase in plankton concentration and decrease in nutrient concentration. Of which process the amplitude of the diurnal variation of concentration often changes 1- to 5-fold, as large as those occurring on annual timescales (Tercier-Waeber et al., 2009; Parker et al., 2010; Nimick et al., 2011). In ecological risk assessment and environmental emergency response, a major concern goes to the short-term variation, e.g. diurnal-timescale, of pollutant concentration. It is important to understand the biochemical and hydraulic processes and patterns controlling diurnal variation. From the experiments and field measurements, the ubiquitous diurnal reaction can exert considerable control on plankton and nutrient concentrations in natural waters. Fluctuation in concentration caused by diurnal reaction may be even larger than concentration decay referring to hydraulic dispersion, which could not be explained by hydraulic dispersion associated with degradation under time-independent reaction rate. In this regard, both the dispersion depending on hydraulic factors and diurnal reaction between plankton and nutrient characterizing biochemical factors have to be taken into account for reliable analysis, monitoring and prediction. Actually the biological and physicochemical effect is extremely complex, however, when we focus on the effect of diurnal reaction on the concentration as well as the critical length and duration of the concentration cloud, some simplified approach with better flexibility for analytical study must be applied.

To investigate the effect of diurnal reaction on the concentration evolution, an instantaneous emission of bicomponent constituents (plankton and nutrient) in an open channel eutrophic flow is theoretically analyzed in this paper, as an effort on hydraulic dispersion with diurnal effect. The specific objectives of this paper are: (I) to formulate the concentration transport process of bicomponent constituents with a time-dependent reaction; (II) to analyze the concentration evolution under the combined action of hydraulic dispersion and diurnal reaction, and (III) to illustrate the evolution of concentration cloud in terms of upper and lower limits of critical length and duration of an influenced region for five typical constituents under given concentration criterions.

2. Formulation

Evolution of plankton cloud in flows is controlled by both the external environmental hydraulics and internal biological effect. In extensive analytical studies in terms of environmental dispersion (Zeng and Chen, 2011; Wu and Chen, 2014; Wang et al., 2013; Wang and Chen, 2015), scattered plankton in the flow is treated as scalar substance. The scalar transport process of plankton associated with ambient conditions including temperature, photocycle, etc. brings variation in plankton mass and concentration (Nimick et al., 2011). Meanwhile, plankton absorbtion, adsorption and assimilation of nutrient lead to changes in the mass and concentration of nutrient. Consequently, the changes in mass and concentration of plankton and nutrient present coupled spontaneity (Sharma et al., 2000). Therefore, the concentration evolution of plankton and nutrient could be described by a set of convection–diffusion-reaction equations.

Consider two constituents transport in a fully developed unidirectional open channel flow with water depth H, in a Cartesian coordinate system with longitudinal x-axis, vertical z-axis, and origin O set at the bed wall, as illustrated in Fig. 1.

The scalar transport process with a time-dependent reaction of first order is given as

$$\frac{\partial S}{\partial t} + u\frac{\partial S}{\partial x} = D\frac{\partial^2 S}{\partial x^2} + D\frac{\partial^2 S}{\partial z^2} + m_1 f_1 S, \tag{1}$$

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} + D\frac{\partial^2 C}{\partial z^2} - m_2 f_2 S, \tag{2}$$

where S (kg m^{-3}) is the concentration of plankton, t (s) the time, u (m s^{-1}) the flow velocity, D (m^2 s^{-1}) the molecular diffusivity, m_1 (s^{-1}) the growth rate constant for plankton (also treated as reaction rate constant), f_1 the time-dependent function for plankton, C (kg m^{-3}) the concentration of nutrient, m_2 (s^{-1}) the reaction rate constant for nutrient, f_2 the time-dependent function for nutrient. $m_2 = m_1/r$, where r is the yield factor for plankton feeding on the nutrient (Hale and Somolinos, 1983), represents the ratio of plankton growth rate to the consumption rate of nutrient. The governing equations belong to a variable-coefficient second-order linear system of parabolic type (Polyanin and Manzhirov, 2006).

Consider the most elementary initial condition in the analytical study as a uniform and instantaneous emission of plankton with mass Q_s and nutrient with mass Q_c at the cross-section of $x = 0$ at time $t = 0$,

$$S(x,z,0) = \frac{Q_s\,\delta(x)}{H}, \quad C(x,z,0) = \frac{Q_c\,\delta(x)}{H}, \tag{3}$$

where Q_s (kg m^{-1}) and Q_c (kg m^{-1}) are mass per depth, and $\delta(x)$ is the Dirac delta function.

The non-penetration conditions at the channel bed of $z = 0$ and the free water surface of $z = H$ read as

$$\left.\frac{\partial S(x,z,t)}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial S(x,z,t)}{\partial z}\right|_{z=H} = 0, \tag{4}$$

$$\left.\frac{\partial C(x,z,t)}{\partial z}\right|_{z=0} = 0, \quad \left.\frac{\partial C(x,z,t)}{\partial z}\right|_{z=H} = 0. \tag{5}$$

Since the amount of discharged plankton and nutrient is finite, concentration boundary conditions at $x = \pm\infty$ for both constituents read as

$$S(\pm\infty,z,t) = 0, \quad C(\pm\infty,z,t) = 0. \tag{6}$$

For brevity, dimensionless parameters are introduced as

$$\begin{gathered} \eta = \frac{x - \bar{u}t}{H}, \quad \zeta = \frac{z}{H}, \quad \tau = \frac{Dt}{H^2}, \\ \Omega_s(\eta,\zeta,\tau) = \frac{S(x,z,t)H^2}{Q_s}, \quad \Omega_c(\eta,\zeta,\tau) = \frac{C(x,z,t)H^2}{Q_c}, \\ \mathrm{Pe} = \frac{\bar{u}H}{D}, \quad \psi = \frac{u-\bar{u}}{\bar{u}} = \frac{3}{2}\zeta(2-\zeta) - 1, \\ M_1 = \frac{H^2 m_1}{D}, \quad M_2 = \frac{M_1}{r}, \end{gathered} \tag{7}$$

where η is the dimensionless longitudinal coordinate moving at the mean flow velocity, ζ the dimensionless depth, τ the dimensionless time, Ω_s and Ω_c the dimensionless concentration for two constituents respectively, Pe the Péclet number represents the relative strength between convection and diffusion, ψ the dimensionless velocity deviation, M_1 and M_2 the dimensionless growth rate constants. The overline indicates the depth-average operation for a variable ϖ as

$$\overline{\varpi} \equiv \int_0^1 \varpi(\zeta)\,\mathrm{d}\zeta. \tag{8}$$