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Total variation diminishing and mass conservative implementation of hydrological flow routing

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SUMMARY

Hydrological flow routing methods are widely used as components of distributed hydrological models and in operational flow forecasting systems. The paper presents a novel approach to reformulate several of these routing schemes as a cascade of implicit pool routing models. Its numerical implementation is mass conservative and total variation diminishing, i.e. the solution does not oscillate or overshoot, for arbitrary time steps. It is shown that these numerical properties are achieved regardless of the accuracy of the scheme and its physical routing characteristics.

Numerical experiments compare the computational performance and accuracy of the novel, reformulated approach with existing schemes including linear reservoir routing, nonlinear reservoir routing, and the Muskingum–Cunge method. We show that the approach can reproduce the original schemes, if these are already mass conservative, otherwise fixes the mass conservation in the reformulated version and improves the solution at sharp gradients by suppressing numerical oscillations, overshooting or negative flows.

1. Introduction

The motivation for this research arises from our use of several hydrological routing schemes in operational flow forecasting and decision support systems primarily for hydropower utilities (Schwanenberg et al., 2014, 2015). In these systems, short-term release decisions at reservoirs are evaluated by routing the flow downstream to other reservoirs or potential inundation areas to access its impact on flood mitigation, hydropower revenues or other objectives. Decisions are derived interactively by system operators or optimization algorithms. A common requirement of both approaches is the need for an accurate, robust and mass-conservative routing scheme.

Existing linear routing schemes such as the linear reservoir routing (Nash, 1958; Kalinin and Miljukov, 1958) or the Muskingum approach (McCarthy, 1938; Kalinin and Miljukov, 1958) are mass conservative by design. However, their linear nature makes it challenging to achieve high accuracy over a broad flow regime due to the assumption of constant wave celerity. Furthermore, sharp gradients in the inflow may lead to unphysical oscillations in the solution and negative flows in particular in the rising limb of a hydrograph (Perumal, 1992; Perumal and Sahoo, 2008). The latter becomes a significant drawback when routing reservoir releases downstream, in particular if hydropower projects implement hydro peaking and generate sharp flow gradients.

Cunge (1969) provides the most popular method to address the accuracy issue by the introduction of the Muskingum–Cunge approach. It makes the former constant parameters of the Muskingum scheme variable and dependent on the flow and other parameters. Successors such as (Price, 1973; Ponce and Chaganti, 1994; Wang et al., 2006) follow this concept and provide variation of the idea. A common feature of all these approaches is the loss of mass conservation. The latter has been subject of a long-lasting analysis and discussion (Tang et al., 1999; Cunge, 2001; Perumal and Sahoo, 2008, and reference therein) up to the paper of Todini (2007) who presents the root cause of the missing mass conservation and a related fix. Other authors such as Perumal and Price (2013) and Reggiani et al. (2014) come to the same conclusions and implement variations of Todini's approach. A common feature of all these schemes is the solution of the nonlinear, variable-parameter Muskingum scheme by an iterative approach. Although these schemes combine accuracy and mass-conservation, the robustness issue is still not adequately addressed.

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Let's leave the field of hydrology and review the progress in other disciplines. The modeling of gas dynamics experienced a breakthrough in its capabilities with the introduction of total variation diminishing (TVD) schemes by Harten (1983). The basic idea behind is the combination of a monotone and non-oscillating first-order scheme with a non-monotone, higher-order accurate scheme such that the blended version keeps the high accuracy in smooth regions of the solutions and reduces to first-order at sharp gradients and shocks by a flux or slope limiter. The interested reader may find a broad review of these methods in Toro (1999) including applications to the shallow water model which has a strong analogy to the Euler equations used in gas dynamics.

The objective of this paper is the formulation of a novel, generalized approach for several hydrological routing schemes including the ones discussed above. The derivation of the approach from a finite volume formulation leads to mass conservation by design regardless of the storage approximation or the physical justification of the routing parameters. Furthermore, we introduce a limiter to receive TVD properties, i.e. numerical oscillations or negative flows will not occur in the solution. Numerical experiments demonstrate the performance of the approach in comparison to existing routing schemes in terms of mass conservation, accuracy and numerical robustness.

2. Methodology

Our starting point is the conservative form of the one-dimensional shallow water or Saint Venant equations provided by

$$\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}=q_L \tag{1}$$

$$\frac{\partial Q}{\partial t}+\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)=gA\left(-\frac{\partial z}{\partial x}-S_f\right) \tag{2}$$

where A is the cross section area, Q is the flow, q_L is a lateral flow, z is the water level, S_f is the friction slope depending on the dimensions space x and time t, g is the acceleration due to gravity.

Following the finite volume methodology, the mass conservation in Eq. (1) can be schematized in the control volume $x=[j-1/2, j+1/2]$, $t=[k-1,k]$ (Fig. 1) by integrating Eq. (1) over space and time according to

$$\int_{k-1}^{k}\int_{j-1/2}^{j+1/2}\left(\frac{\partial A}{\partial t}+\frac{\partial Q}{\partial x}\right)dx\,dt=\int_{k-1}^{k}\int_{j-1/2}^{j+1/2}q_L\,dx\,dt \tag{3}$$

leading to

$$\begin{aligned}&S_j^k-S_j^{k-1}-\Delta t\left(Q_{j-1/2}^{k-1/2}-Q_{j+1/2}^{k-1/2}\right)=\Delta t\Delta x q_{L,j}^{k-1/2}\\&\text{with } S_j^{k-1,k}=\textstyle\int_{j-1/2}^{j+1/2}A^{k-1,k}dx,\ Q_{j\pm1/2}^{k-1/2}=\int_{k-1}^{k}Q_{j\pm1/2}dt,\ q_{L,j}^{k-1/2}=\int_{k-1}^{k}\int_{j-1/2}^{j+1/2}q_L\end{aligned} \tag{4}$$

where the storage S is the integrated area A along the spatial dimension of the control volume $x\in[j-1/2, j+1/2]$. An important aspect of the schematization is that the quantities $S_j^{k-1,k}$, $Q_{j\pm1/2}^{k-1/2}$ and $q_{L,j}^{k-1/2}$ are integrals over the boundaries of the control volume and not a representation at a specific point as in a finite difference approach. $S_j^{k-1,k}$ is the quantity we want to conserve, i.e. the volume of water in the system. $Q_{j\pm1/2}^{k-1/2}$ is the so-called flux term which defines the exchange of water between the finite volumes. If the flux has the same unit of the quantity to be conserved, it is obvious that such a formulation is conservative by definition.

Up to this point, our derivation has been generic for hydraulic or hydrological routing methods. Let's now introduce the two main ideas behind hydrological routing:

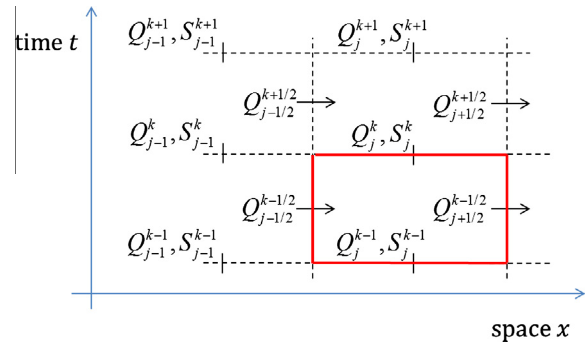

Fig. 1. Control volumes formulation of the mass balance equation where the indices j and k denote discrete variables in space x and time t, respectively.

1. Under the assumption of a sufficiently steep river reach and neglecting backwater effects, the flux $Q_{j-1/2}$ is replaced by its upstream value according to $Q_{j-1/2}=Q_{j-1}$. This corresponds to a first-order upwind scheme. It decouples the computation of the finite volumes in a time step and enables a subsequent computation from upstream to downstream. In contrary, a hydraulic model depends on the simultaneous computation of all volumes at one time step. This fact is the key to the higher computational performance of hydrological routing approaches.
2. The momentum equation Eq. (2) gets replaces by an algebraic equation to define a relation between storage, flow and some other parameters according to a general implicit function $f(S,Q,p)=0$.

For simplicity, we further assume that the storage can get expressed as an explicit function of flow and some parameters according to $S(Q,p)$. This allows rewriting Eq. (4) for the control volume j to receive

$$\frac{S^k(I^k,Q^k,p)-S^{k-1}(I^{k-1},Q^{k-1},p)}{\Delta t}-I^{k-1/2}+Q^{k-1/2}=0 \tag{5}$$

where we introduce a simplified notation according to $I=Q_{j-1}+\Delta x q_{L,j}$, $Q=Q_j$ and $S=S_j$. The continuous form of Eq. (5) is the one of a lumped nonlinear reservoir according to the ordinary differential equation (ODE)

$$\frac{dS(I,Q,p)}{dt}-I+Q=0 \tag{6}$$

On the other hand, we receive a discrete-time form of Eq. (5) by the application of the θ-method to express the intermediates $I^{k-1/2}$, $Q^{k-1/2}$ as variables of the time steps $k-1,k$ by

$$\begin{aligned}I^{k-1/2}&=(1-\theta_I)I^{k-1}+\theta_I I^k\\Q^{k-1/2}&=(1-\theta_Q)Q^{k-1}+\theta_Q Q^k\end{aligned} \tag{7}$$

to receive

$$\begin{aligned}F(I^{k-1,k},Q^{k-1,k})=&\frac{S^k(I^k,Q^k,p)-S^{k-1}(I^{k-1},Q^{k-1},p)}{\Delta t}\\&-(1-\theta_I)I^{k-1}-\theta_I I^k+(1-\theta_Q)Q^{k-1}+\theta_Q Q^k=0\end{aligned} \tag{8}$$

where F is an implicit function representing the mass error in the reservoir and θ_I, θ_Q are time weighting coefficients with unconditional stability in the range $[0.5, 1]$. Eq. (8) is either linear or nonlinear depending on the storage function $S(I,Q,p)$. In a generic setup, the solution of $F=0$ can be achieved by a Newton–Raphson iteration procedure. Since the inflow is given at both time steps $k-1,k$ and the outflow is available at $k-1$, the only