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Analytical solutions of three-dimensional groundwater flow to a well in a leaky sloping fault-zone aquifer

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SUMMARY

A semi-analytical solution was presented for groundwater flow due to pumping in a leaky sloping fault-zone aquifer surrounded by permeable matrices. The flow in the aquifer was descried by a three-dimensional flow equation, and the flow in the upper and lower matrix blocks are described by a one-dimensional flow equation. A first-order free-water surface equation at the outcrop of the fault-zone aquifer was used to describe the water table condition. The Laplace domain solution was derived using Laplace transform and finite Fourier transform techniques and the semi-analytical solutions in the real time domain were evaluated using the numerical inverse Laplace transform method. The solution was in excellent agreement with Theis solution combined with superposition principle as well as the solution of Huang et al. (2014). It was found that the drawdown increases as the sloping angle of the aquifer increases in early time and the impact of the angle is insignificant after pumping for a long time. The free-water surface boundary as additional source recharges the fault aquifer and significantly affect the drawdown at later time. The surrounding permeable matrices have a strong influence on drawdown but this influence can be neglected when the ratio of the specific storage and the ratio of the hydraulic conductivity of the matrices to those of the fault aquifer are less than 0.001.

1. Introduction

Fault-zone aquifers are widespread globally (Babiker and Gudmundsson, 2004; Holland, 2012; Mabee et al., 1994) and not well understood in terms of groundwater hydraulics (Bense et al., 2013). Water movement through fault zones needs to be considered in many engineering applications, e.g., water supply (Haneberg, 1995; Huntoon, 1979), waste repository construction (Babiker and Gudmundsson, 2004; Bredehoeft, 1997), underground mining and petroleum exploitation (Fowles and Burley, 1994; Harris et al., 2002; Zhang et al., 2003). Antonio and Pacheco (2002) studied the geometry of fault zones and concluded that most fault zones have certain incline angles. The pattern of groundwater flow in a sloping fault-zone aquifer due to pumping was influenced greatly by the angle of the slope (Liu et al., 2015) and by leakage from the surrounding rock matrices on both sides of the fault zone (Anderson, 2006; Antonio and Pacheco, 2002).

Several analytical solutions for groundwater flow due to well pumping in sloping aquifers have been developed in previous studies. Hantush (1962) obtained an analytical solution for the hydraulic head in a wedge-shaped aquifer and found that the flow in the aquifer cannot be estimated by the flow in an aquifer of uniform thickness. Hantush (1964) presented a two-dimensional solution to the steady-state groundwater flow due to well pumping in a sloping unconfined aquifer resting on a semi-permeable layer that overlay on an artesian aquifer. Antonio and Pacheco (2002) used the Cooper–Jacob method (Cooper and Jacob, 1946) to describe the drawdown in a sloping fault-zone aquifer which extends semi-infinitely from the outcrop at which water table was presented by a free surface. Zhan and Zlotnik (2002) obtained analytical solutions for flow to a slant well in horizontal unconfined aquifers considering both instantaneous drainage and delayed yield. Huang et al. (2014) developed a two-dimensional solution for drawdown induced by pumping in a sloping fault-zone aquifer with the water table at the outcrop.

None of above-mentioned studies considered the three-dimensional groundwater flow to a pumping well in a sloping fault-zone aquifer as well as the leakage from both sides of the surrounding matrix blocks, which is the primary objective of this study. In this paper we employed the Laplace-finite Fourier transform to derive the solution for this case. In the following we will

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first describe the mathematical model and solutions, then present the results and discussion, and finally summarize our study and draw some conclusions.

2. Formulation

We consider the three-dimensional groundwater flow to a pumping well in a sloping fault-zone aquifer embedded in surrounding rock which are commonly represented as low permeability matrices (Fig. 1). The fault-zone aquifer extends finitely from the outcrop at the ground surface where the groundwater table is represented as a free-water surface for the aquifer which thus is an unconfined aquifer. The symbols θ and B are for the sloping angle and the thickness of aquifer, respectively. The y axis is horizontal, and the x and z axes are parallel and perpendicular to the aquifer, respectively, in the sloping coordinate system (Fig. 1). The length of the aquifer along the x-coordinate is W. The upper and lower matrix contact with the fault-zone aquifer at $z = B$, and 0, respectively. We defined the x', y', and z' as the horizontal and vertical coordinate system (Fig. 1). The free-water surface at the outcrop is represented by linearized kinematic water table equation (Moench, 1995; Neuman, 1972) as following

$$K_{z'}\frac{\partial s}{\partial z'} + S_y\frac{\partial s}{\partial t} = 0, \quad \text{at } z' = W\sin\theta - s \quad \text{and} \quad x'_l \leqslant x' \leqslant x'_r \tag{1}$$

where $K_{z'}$ is the vertically hydraulic conductivity, S_y is the specific yield, s is the change in the drawdown from the initial static level in the aquifer due to well pumping, $x'_l = W\cos\theta - B/(\sin\theta)$, and $x'_r = W\cos\theta$. The relationship between the Cartesian and the sloping coordinate systems are written as

$$x = x'\cos\theta + z'\sin\theta \tag{2a}$$

$$y = y' \tag{2b}$$

$$z = -x'\sin\theta + z'\cos\theta \tag{2c}$$

Substituting Eq. (2) into Eq. (1), one obtains

$$K_{z'}\left(\frac{\partial s}{\partial x}\frac{\partial x}{\partial z'} + \frac{\partial s}{\partial z}\frac{\partial z}{\partial z'}\right) + S_y\frac{\partial s}{\partial t} = 0 \tag{3}$$

where $\partial x/\partial z' = \sin\theta$. The derivative term $\partial s/\partial z$ approaches zero since groundwater moves along the x-direction at the free-water surface. Thus Eq. (1) can be transformed to the sloping coordinate system as following (Huang et al., 2014)

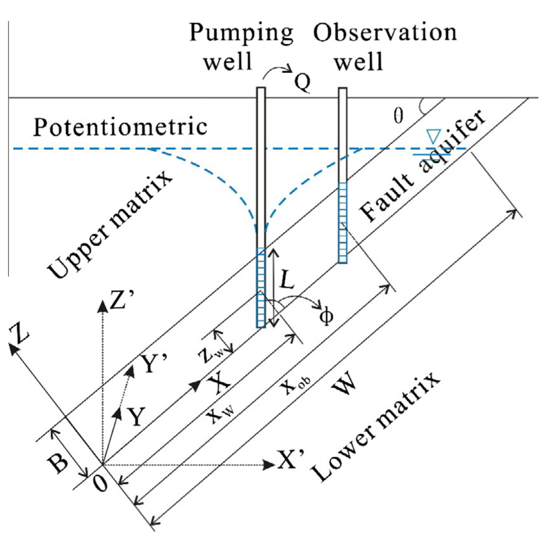

Fig. 1. Schematic diagram of a pumping well and an observation well in a sloping fault zone aquifer with leakage from upper and lower matrices.

$$K_{z'}\sin\theta\frac{\partial s}{\partial x} + S_y\frac{\partial s}{\partial t} = 0 \quad \text{at } x = W - s/\sin\theta \tag{4}$$

where $W - s/\sin\theta$ represents a water table position in x-direction. Eq. (4) is a nonlinear equation due to the presence of the unknown water table position. In this study we neglect the term $s/\sin\theta$ by assuming that drawdown is very small compared with the aquifer thickness (Huang et al., 2014). Thus, Eq. (4) can be linearized by fixing the free surface boundary as

$$K_{z'}\sin\theta\frac{\partial s}{\partial x} + S_y\frac{\partial s}{\partial t} = 0 \quad \text{at } x = W \tag{5}$$

The governing equation for the drawdown (s) for the three-dimensional transient flow in response to a constant pumping rate Q at a point (x_0, y_0, z_0) inside the aquifer is written as

$$K_x\frac{\partial^2 s}{\partial x^2} + K_y\frac{\partial^2 s}{\partial y^2} + K_z\frac{\partial^2 s}{\partial z^2} - Q\delta(x - x_0)\delta(y - y_0)\delta(z - z_0) = S_S\frac{\partial s}{\partial t} \tag{6a}$$

which is subject to the following initial and boundary conditions:

$$s(x, y, z, 0) = 0 \tag{6b}$$

$$\frac{\partial s}{\partial x}(x, y, z, t)|_{x=0} = 0 \tag{6c}$$

$$\lim_{y\to\pm\infty} s(x, y, z, t) = 0 \tag{6d}$$

$$K_{z'}\sin\theta\frac{\partial s}{\partial x} + S_y\frac{\partial s}{\partial t} = 0 \quad \text{at } x = W \tag{6e}$$

where K_x, K_y, K_z are the aquifer hydraulic conductivity in x-, y-, and z-direction, respectively, S_S is the specific storage, Q is a pumping rate, and δ is the Dirac delta function.

Usually, the permeability of a fault is much larger than that of the matrix (Babiker and Gudmundsson, 2004; Bauer et al., 2015; Gudmundsson et al., 2001; Micarelli et al., 2006). The flow component in the direction of perpendicular to the fault/matrix interface in matrix is dominant when pumping water in the fault. Thus in this study the groundwater in upper and low matrix is assumed as semi-infinitely one-dimensional flow along z-direction in response to the pumping in the fault aquifer. The groundwater flow between the matrices and the fault-zone aquifer are continuous by equal drawdown and normal flux at the interface. The governing equation and corresponding initial and boundaries conditions for the groundwater flow in the upper matrix are written as

$$K_{ZU}\frac{\partial^2 s_U}{\partial z^2} = S_{SU}\frac{\partial s_U}{\partial t}, B \leqslant z < +\infty \tag{7a}$$

$$s_U(z, 0) = 0 \tag{7b}$$

$$s_U(z, t)|_{z=+\infty} = 0 \tag{7c}$$

$$K_{ZU}\frac{\partial s_U}{\partial z}(z, t)|_{z=B} = K_z\frac{\partial s}{\partial z}(x, y, z, t)|_{z=B} \tag{7d}$$

$$s_U(z, t)|_{z=B} = s(x, y, z, t)|_{z=B} \tag{7e}$$

where s_U is the drawdown in the upper matrix, K_{ZU} is the hydraulic conductivity in z-direction in the upper matrix, S_{SU} is the specific storage of the upper matrix.

The governing equation of groundwater and the correspondingly initial and boundaries condition for the groundwater flow in the lower matrix are given by

$$K_{zL}\frac{\partial^2 s_L}{\partial z^2} = S_{SL}\frac{\partial s_L}{\partial t}, \quad -\infty \leqslant z < 0 \tag{8a}$$

$$s_L(z, 0) = 0 \tag{8b}$$

$$s_U(z, t)|_{z=-\infty} = 0 \tag{8c}$$

$$K_{zL}\frac{\partial s_L}{\partial z}(z, t)|_{z=0} = K_z\frac{\partial s}{\partial z}(x, y, z, t)|_{z=0} \tag{8d}$$

$$s_L(z, t)|_{z=0} = s(x, y, z, t)|_{z=0} \tag{8e}$$