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A second-order treatment to the wet–dry interface of shallow water

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SUMMARY

The paper presents a second-order lattice Boltzmann method (LBM) for treating the wet–dry interface of shallow water flows. This approach is improved according to the Chapman–Enskog analysis and the Taylor expansion, which are used to set up the relation of the dry cell and its adjacent wet cell. The external forces, such as bed friction and wind stress, are directly included in the wetting and drying boundary treatment. However, the viscous effect cannot be absolutely removed, as the single relaxation time, $\tau > 0.5$, should be retained for the reason of stability. In order to verify the scheme, two one-dimensional (1D) and one two-dimensional (2D) numerical cases are carried out. The results indicate that the approach is superior to the first-order scheme, comparing with the other numerical solutions and the experimental data. It can be concluded that the proposed scheme is more accurate and effective in simulating shallow water fronts.

1. Introduction

Shallow water flows widely exist in nature, such as open channels, rivers, and estuaries. Such flows have the same characteristic that the vertical scale is ordinarily much smaller compared with the horizontal scale. Ignoring the vertical acceleration and under the assumption of hydrostatic pressure, the shallow water equations can be derived from the incompressible Navier–Stokes equations. In order to solve these equations, many numerical methods like the finite difference method (Casulli, 1990), the finite volume method (Hu et al., 2000) and the finite element method (Leclerc et al., 1990), have been proposed. Unlike the traditional numerical approaches, which solve the shallow water equations based on the direct discretizations of these equations, the lattice Boltzmann method (LBM) lies in the description of the macroscopical fluid flows from the microscopic flow behavior through particle distribution functions. The LBM, originated from the lattice gas automata (Benzi et al., 1992) on the strength of easy programming, inherent parallel features and effective treatment for complex boundary conditions, has developed into an alternative approach for simulating shallow water flows (Chen and Doolen, 1998; Zhou, 2002; Dellar, 2002; Liu et al., 2010). In recent years, the LBM has been successfully used for simulating a variety of problems. O'Brien et al. (2002) used a modified lattice Boltzmann scheme for reactive transport in porous media. Zhang et al. (2005) presented a pore-scale modeling of soil hydraulic conductivity using the LBM and thin-section technique. Tubbs and Tsai (2009) conducted the parallel computation for multi-layer shallow water flows. Liu et al. (2015) developed the LBM for the Saint–Venant equations.

Generally, the shallow water flow involves complex phenomena, such as wave run-up and wave overtopping, which often lead to wetting and drying boundary problem. It is easy to recognize that with the movement of the wet–dry interface the computational domain changes constantly, which affects the accuracy of calculation. Therefore, wet–dry boundary conditions have received much attention. In the existing models, many kinds of wet and dry processing methods were put forward. Madsen et al. (1997) employed the permeable slot method (Tao, 1984), which tiled the narrow water to shore beach based on the principle of water balance. Sleigh et al. (1998) utilized an approximate Riemann solver to determine flow directionality in conjunction with an effective means of dealing with wetting and drying at the boundaries. Kennedy et al. (2000) improved the slot method, which reduced the loss of water for populating the slit. Lynett et al. (2002) presented a linear extrapolation scheme, which is a pure mathematical method. van't Hof and Vollebregt (2005) used the artificial porosity method for wetting and drying in shallow water

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flow. Frandsen (2008) incorporated the thin film and the liner extrapolation scheme to treat the wet–dry interface.

Even though there are many wet–dry treating methods, most of them did not physically consider the external force terms. Clearly, external forces such as bed friction and wind stress are very important in simulations, which will influence the boundary movement that directly affects the accuracy of the solution. Buick and Greated (1999) considered the gravity using a lattice Boltzmann model. Liu and Zhou (2014) proposed a new lattice Boltzmann approach to simulating wetting and drying processes in shallow water flows, straightly including external forces in simulation. However, the treatment of the wet–dry interface in the approach is only first-order accurate due to using first-order non-equilibrium part of particle distribution function. Therefore, the objective of the present study is to further improve it to second-order accuracy. Two benchmark tests are carried out, and the results are particularly favorable compared with either experimental data or the first-order solutions. This paper is organized as follows: Section 2 introduces the lattice Boltzmann model and the treatment of the wet–dry boundary; Section 3 presents the verification and application of the model; Section 4 gives conclusions.

2. Theoretical background

2.1. Lattice Boltzmann model for shallow water equations

The lattice Boltzmann method involves two steps, i.e. the streaming step and the collision step. In the streaming step, the particles move to the neighboring lattice points in their velocities governed by (Zhou, 2004)

$$f_\alpha(\mathbf{x}+\mathbf{e}_\alpha\Delta t, t+\Delta t) = f'_\alpha(\mathbf{x},t) + W_\alpha \frac{\Delta t}{C_s^2} e_{\alpha i}F_i(x,t) \tag{1}$$

where f_α is the particle distribution function; f'_α is the value of f_α before the streaming; $e = \Delta x/\Delta t$; Δx is the lattice size; F_i is the force term in the i direction; Δt is the time step; for the one-dimensional (1D) based on D1Q3 lattice (see Fig. 1a), $e_0 = 0$, $e_1 = e$ and $e_2 = -e$; and for the two-dimensional (2D) based on D2Q9 lattice (see Fig. 1b), e_α is defined as Table 1; C_s is the local sound speed, given as Eq. (2); W_α is the weight coefficient, being 1/4 for D1Q3 lattice, and is shown as Eq. (3) for D2Q9 lattice.

$$C_s = \begin{cases} \frac{e}{\sqrt{2}}, & \text{D1Q3;} \\ \frac{e}{\sqrt{3}}, & \text{D2Q9;} \end{cases} \tag{2}$$

Table 1
The velocity vector for D2Q9 lattice.

α	0	1	2	3	4	5	6	7	8
$e_{\alpha x}$	0	e	e	0	$-e$	$-e$	$-e$	0	e
$e_{\alpha y}$	0	0	e	e	e	0	$-e$	$-e$	$-e$

$$W_\alpha = \begin{cases} \frac{1}{9}, & \alpha = 1,3,5,7 \\ \frac{1}{36}, & \alpha = 2,4,6,8 \end{cases} \tag{3}$$

In the collision step, f'_α can be written as

$$f'_\alpha(\mathbf{x},t) = f_\alpha(\mathbf{x},t) - \frac{1}{\tau}(f_\alpha - f_\alpha^{eq}), \tag{4}$$

where f_α^{eq} is the local equilibrium distribution function, τ is the single relaxation time. If f_α^{eq} for D1Q3 lattice is written as

$$f_\alpha^{eq} = \begin{cases} h - \frac{hu^2}{e^2} - \frac{gh^2}{2e^2}, & \alpha = 0, \\ \frac{gh^2}{4e^2} + \frac{hu^2}{2e^2} + \frac{hu}{2e}, & \alpha = 1, \\ \frac{gh^2}{4e^2} + \frac{hu^2}{2e^2} - \frac{hu}{2e}, & \alpha = 2, \end{cases} \tag{5}$$

and for D2Q9 lattice

$$f_\alpha^{eq} = \begin{cases} h - \frac{5gh^2}{6e^2} - \frac{2h}{3e^2}u_iu_i, & \alpha = 0 \\ \frac{gh^2}{6e^2} + \frac{h}{3e^2}e_{\alpha i}u_i + \frac{h}{2e^4}e_{\alpha i}e_{\alpha j}u_iu_j - \frac{h}{6e^2}u_iu_i, & \alpha = 1,3,5,7 \\ \frac{gh^2}{24e^2} + \frac{h}{12e^2}e_{\alpha i}u_i + \frac{h}{8e^4}e_{\alpha i}e_{\alpha j}u_iu_j - \frac{h}{24e^2}u_iu_i, & \alpha = 2,4,6,8 \end{cases} \tag{6}$$

the nonlinear shallow water Eqs. (7) and (8) can be recovered by using the Chapman–Enskog procedure (Zhou, 2004).

$$\frac{\partial h}{\partial t} + \frac{\partial (hu_j)}{\partial x_j} = 0, \tag{7}$$

$$\frac{\partial (hu_i)}{\partial t} + \frac{\partial (hu_iu_j)}{\partial x_j} = -\frac{g}{2}\frac{\partial h^2}{\partial x_i} + \nu\frac{\partial^2 (hu_i)}{\partial x_j \partial x_j} + F_i, \tag{8}$$

In the above equations, the Cartesian coordinate system and the Einstein summation convention over Latin indices are used; ν is the kinematic viscosity of water defined as

$$\nu = \begin{cases} e^2\Delta t(\tau - \frac{1}{2}), & \text{D1Q3;} \\ \frac{e^2\Delta t}{3}(\tau - \frac{1}{2}), & \text{D2Q9;} \end{cases} \tag{9}$$

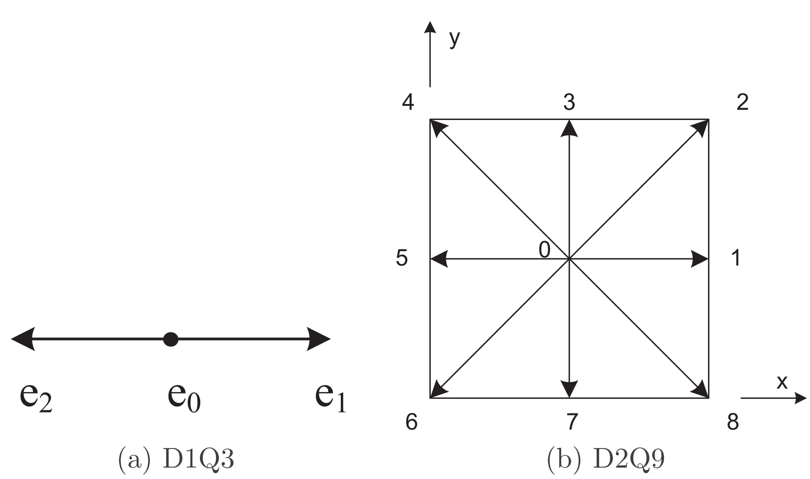

Fig. 1. Lattice patterns.