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Development of one-dimensional solutions for water infiltration. Analysis and parameters estimation

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SUMMARY

The design of an irrigation system requires proper understanding of the infiltration process. Two original approximate solutions were developed in this study in order to determine one-dimensional vertical infiltration and profile water distribution.

The Boltzmann transformation was used to solve the Richards equation. The accuracy of the solutions was verified by comparing results obtained with Hydrus-1D simulations. The solutions were also compared with the US Department of Agriculture-Natural Resources and Conservation Service (USDA-NRCS) intake families for infiltrated depths between 2 and 16 cm, using soils from Carsel and Parrish database.

Simulations results of both solutions were in close agreement with Hydrus-1D. For all the cases studied, the corresponding root mean square error (RMSE) values were up to 0.56 cm, with coefficients of correlation (R^2) larger than 0.99.

Therefore, The comparison shows inexact matching, with acceptable correspondence, between simulated soils and NRCS curve numbers, The procedure developed to fit the USDA–NRCS intake families with the proposed solutions leads to characterize each family by the van Genuchten–Mualem parameters, and an initial soil water content. Furthermore, results suggests to consider initial water content in fine-textured soils. In the case of intake families, initial water content began to affect in Family 0.5 and became very decisive in the Family 0.05.

Additionally, the proposed method, which are applicable to any descriptor of unsaturated hydraulic properties, have practical application, including accurate determination of the sorptivity of a soil at various water contents, and requires low computational effort.

1. Introduction

Infiltration is a process of great practical importance to irrigation design, particularly in surface irrigation. Empirical equations, such as Kostiakov, Kostiakov-Lewis or USDA-NRCS intake families (USDA, 1974, 2009; Walker et al., 2006) are the most commonly used infiltration equations in surface irrigation studies. The parameters, with no physical meaning, are estimated from experimental data, and they are only valid for the test conditions.

Infiltration can be treated theoretically by solving Richards' equation with numerical simulation models, such as Hydrus-1D (Šimùnek et al., 2005). The use of numerical methods has been proved accurate and can practically deal with all initial and boundary conditions. However, their practical application, in some cases, is limited because they are too complex. It requires more parameterization and may occasionally experience convergence problems (Celia et al., 1990; Clemente et al., 1994; Vogel et al., 2001; Schaap and van Genuchten, 2006). A first estimation of these parameters can be achieved directly from soil textural classification by pedotransfer functions (Schaap et al., 1998), although it needs to be carefully optimized against pre-existing observed data to obtain good simulation results (Bohne et al., 1995; Feddes et al., 1988).

As an intermediate situation, approximate solutions, with physically based approaches, have often been used for modeling infiltration and soil water redistribution (Smith et al., 1993; Corradini et al., 1997; Jury and Horton, 2004; Parlange et al., 2002; Barry et al., 2012 among others). The formulations were shown to perform satisfactorily when compared with a numerical solution of the Richards equation. For the case of infiltration,

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the use of some expression is conditioned by an accurate determination of parameters S, and K_s. Significant research has been performed to evaluate those parameters (Brutsaert, 1977; Parlange et al., 1982; Haverkamp et al., 1994; Smith, 2002; Warrick, 2003). However, most of the expressions developed to estimate sorptivity are only valid for dry soils. The procedures used needs several calculations steps, and are too cumbersome for practical application. In many cases, only experts can obtain an adequate result.

Therefore, simplified but carefully designed approaches are usually required (Corradini et al., 1997). These can be potentially used to provide similar results in details and accuracy to models and avoid their disadvantages.

The main objective of this paper is to propose a simplified approach in order to determine the vertical infiltration and distribution of water along the soil profile, $\theta(z,t)$. The method will be described in detail, and compared with numerical simulations of Hydrus-1D for various soil types. The method will also be used to estimate the soil hydraulic parameters for the USDA-NRCS intake families, and to evaluate the effect of initial water content on the infiltration process.

2. Material and methods

2.1. Infiltration theory

A variety of particular analytical solutions of Richards equation have been developed, with the common assumptions of homogeneous soil profile and initially ponded condition (e.g., Green and Ampt, 1911; Philip, 1957; Mein and Larson, 1973; Parlange, 1975; Parlange et al., 1982; Swartzendruber and Clague, 1989).

Green and Ampt equation, GA (Green and Ampt, 1911) for prediction of infiltration phenomena is still used and has been subject to considerable attention and different modifications (Smith et al., 1993; Corradini et al., 1997; Swamee et al., 2012 among others). GA equation, assuming that depth of ponded water on the surface is negligible, relates cumulative infiltration, i_a, to the infiltration rate, i_i, or the time, t, as follows:

$$i_i = \frac{di_a}{dt} = K_s \cdot \left(1 + \frac{h_f \cdot \Delta\theta}{i_a}\right) \rightarrow t = \frac{i_a}{K_s} - \frac{h_f \cdot \Delta\theta}{K_s} \cdot \ln\left(\frac{i_a}{h_f \cdot \Delta\theta} + 1\right) \quad (1)$$

where K_s [LT^{-1}] is the saturated hydraulic conductivity, h_f [L] is the capillary tension head at the wetted front, and $\Delta\theta = \theta_s - \theta_0$ [–] is the difference between the saturated and initial moisture contents.

The $i_a(t)$ is an implicit function; however, it can be easily determined by successive iterations. It can be seen that Eq. (1) depends on parameters, K_s and $h_f \cdot \Delta\theta$. While the first parameter is unquestionable, the rationality of the second is questionable because h_f depends on θ_0 and changes during the irrigation event.

Philip (1957, 1969) solved Richards' equation, under the conditions of constant soil moisture content at the surface into a semi-infinite soil profile, giving i_a as the sum of a series of terms of ascending powers of $t^{1/2}$ in the form

$$i_a = \sqrt{t} \cdot \int_{\theta_0}^{\theta_s} u(\theta) \cdot d\theta + t \cdot \int_{\theta_0}^{\theta_s} \chi(\theta) \cdot d\theta + t^{3/2} \cdot \int_{\theta_0}^{\theta_s} \varphi(\theta) \cdot d\theta + \cdots \cong S \cdot \sqrt{t} + E \cdot t \cong S \cdot \sqrt{t} + K_s \cdot t \quad (2)$$

where S is sorptivity, and E is the saturated hydraulic conductivity $E \cong K_s$.

Philip's power series solution is still often applied. However, it basically describes well the infiltration only for short to intermediate times. Generally, the equation is truncated so it only contains two terms. The first term, $S \cdot \sqrt{t}$, is used to describe horizontal infiltration, when the gravity can be neglected, and the second term, $E \cdot t$, represents the infiltration due to the downward force of gravity. Usually, the parameter E is considered roughly identical to K_s. Programs to evaluate the first, second and third terms can be found in Warrick (2003).

Applying the same assumption adopted in the determination of GA equation for horizontal infiltration and equaling to the first term of Philip's solution, the following relationship was obtained between the three parameters previously considered (two linearly independent).

$$S = \sqrt{2K_s \cdot \Delta\theta \cdot h_f} \quad (3)$$

The parameter, S is defined for horizontal infiltration and its use in irrigation seems to be more adequate than $h_f \cdot \Delta\theta$. However, in case of considering h_f as a constant, as proposed by Rawls et al. (1993), the evaluation of θ_0 effect would be simplified.

Considering Eq. (3), Eq. (1) can be expressed in terms of K_s and S, resulting in:

$$t = \frac{S^2}{2K_s^2} \cdot \left(\frac{i_a \cdot 2K_s}{S^2} - \ln\left(\frac{i_a \cdot 2K_s}{S^2} + 1\right)\right) \quad (4)$$

Simplified Eqs. (2) and (4) have been written in dimensionless form, i_a^* ($t^* = t/T_e, i_a^* = i_a/I_{ae}$) by introducing the scales, $T_e = S^2/K_s^2$ and $I_{ae} = T_e \cdot K_s$, as follows:

$$\text{PS, Philip (with } E = K_s) : i_a^* = \sqrt{t^*} + t^* \quad (5)$$

$$\text{GA, Green–Ampt} : t^* = i_a^* - \frac{1}{2} \cdot \ln\left(2i_a^* + 1\right) \quad (6)$$

Parlange (1975) proposed an equation for cumulative infiltration expressed in terms of Philip's model parameters. Swartzendruber and Clague (1989) modified Parlange's equation, introducing a coefficient, a, making it more general as:

$$i_a^* = t^* + \frac{1}{a} \cdot \left[1 - \exp\left(-a \cdot \sqrt{t^*}\right)\right] \quad (7)$$

The use of a = 2 leads to Parlange's equation. With a = 0.5 is an equation close to GA, and with $a \cong 0$ and $a > 0$, is an equally close approximation for PS. As a result, Eq. (7) could be considered a general form of the commented equations.

Fig. 1 shows dimensionless cumulative infiltration as a function of t^*, calculated by PS, GA and by numerical integration of Richards' equation and using the parameters showed in Table 1 (for sandy,

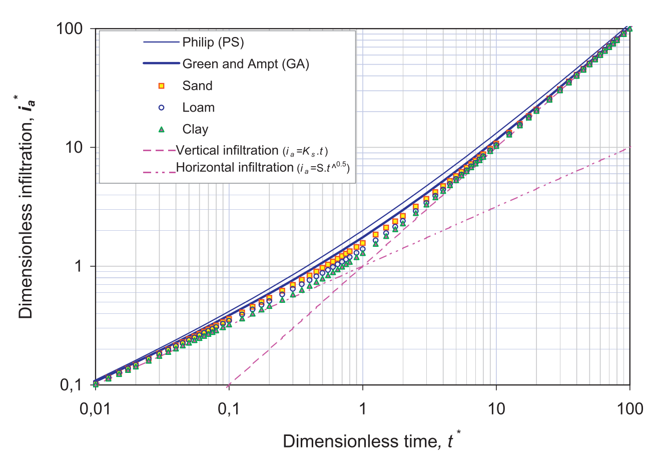

Fig. 1. Dimensionless infiltration, i_a^*, as a function of dimensionless time, t^*, calculated using Philips (PS) and Green and Ampt (GA) simplified expressions, and approximate values obtained by numerical integration for Sand, Loam and Clay soils of Carsel and Parrish database (1988) using van Genuchten–Mualem model and the parameters of Table 1.