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An explicit approach to capture diffusive effects in finite water-content method for solving vadose zone flow

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SUMMARY

Vadose zone flow problems are usually solved from the Richards equation. Solution to the Richards equation is generally challenging because the hydraulic conductivity and diffusivity in the equation are strongly non-linear functions of water content. The finite water-content method was proposed as an alternative general solution method of the vadose zone flow problem for infiltration, falling slugs, and vadose zone response to water table dynamics based on discretizing the water content domain into numerous bins instead of the traditional spatial discretization. In this study, we develop an improved approach to the original finite water-content method (referred to as TO method hereinafter) that better simulates diffusive effects but retains the robustness of the TO method. The approach treats advection and diffusion separately and considers diffusion on a bin by bin basis. After discretizing into water content bins, we treat the conductivity and diffusivity in individual bins as water content dependent constant evaluated at given water content corresponding to each bin. For each bin, we can solve the flow equations analytically since the hydraulic conductivity and diffusivity can be treated as a constant. We then develop solutions for each bin to determine the diffusive water amounts at each time step. The water amount ahead of the convective front for each bin is redistributed among water content bins to account for diffusive effects. The application of developed solution is straightforward only involving algebraic manipulations at each time step. The method can mainly improve water content profiles, but has no significant difference for the total infiltration rate and cumulative infiltration compared to the TO method. Although the method separately deals with advection and diffusion, it can account for the coupling effects of advection and diffusion reasonably well.

1. Introduction

Quantifying flow of water through vadose zone soils is required for a large number of hydrological applications. Flow in unsaturated soils is often modeled using the Richards equation (RE) (Richards, 1931) and closed by hydraulic property functions to describe the relationship among pressures, saturations, and hydraulic conductivities (Caviedes-Voullieme et al., 2013; Liang and Uchida, 2014). The Richards equation is difficult to solve both analytically and numerically due to its parabolic form in combination with the strong non-linearity of the soil hydraulic functions (e.g., Brooks and Corey, 1964; van Genuchten, 1980; Vogel et al., 2001). Abrupt changes of water content in steep wetting fronts in dry soils, or drying fronts in wet soils, may pose challenging problems in numerical solutions of the Richards equation (Celia et al., 1990; Miller et al., 1998; van Dam and Feddes, 2000) in terms of the accuracy, stability, and rate of convergence of the numerical algorithms.

An alternative finite water-content method to the one-dimensional partial differential equation (PDE) for unsaturated porous media flow attributed to Richards (1931) was first proposed by Talbot and Ogden (2008). Recently a much improved and complete method was cast in a more rigorous physical and mathematical fashion (Ogden et al., 2015c). The finite water-content method (referred to as TO method hereinafter) is a general solution method of the vadose zone flow problem for infiltration, falling slugs, and vadose zone response to water table dynamics in an unsaturated porous medium (e.g., Ogden et al., 2015b). The method represents a significant advance to overcome many issues related to numerically solving the Richards equation in that it can simulate sharp fronts, and is guaranteed to converge using a finite-volume

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solution. The method can be easily incorporated in simulations of large watersheds or in regional climate-hydrology models where solving the RE at millions of nodes is required without the risk of non-convergence often encountered in the numerical solutions of the Richards equation, which often jeopardizes the stability of the entire model simulation (Ogden et al., 2015c). The fact that the finite water content method is arithmetic and an explicit solution of ordinary differential equation suggests that it will be amenable to significant improvements in computational efficiency. Therefore, the method is particularly useful in applications such as high-resolution quasi-3D simulations of large-scale climate and hydrology problems (Ogden et al., 2015a).

A distinguishing characteristic of the TO method is the discretization of the water content domain into “bins” of water content $\Delta\theta$, leaving the vertical spatial dimension as a continuum over which the model can freely move wetting and drying fronts. The TO method is advective, driven by gravity and capillary gradients, and without an explicit representation of soil water diffusivity (Ogden et al., 2015c). This is similar to the idea behind the Green–Ampt equation (Green and Ampt, 1911), but it is considered for each individual water content bin.

The main objective of this study is to incorporate soil water diffusivity into the TO method to improve water content profiles. A key concept in the proposed method is to consider diffusive effects on the basis of individual water content bins. Three new ideas are integrated in the analytical developments as described below. First, we solve water content advection–diffusion equation to develop water content profile for each bin. The velocity term in the equation is similar to the TO method. Since the water content domain is discretized into numerous bins, the diffusivity for the bin is approximated as a constant evaluated at the water content for that bin and the wetting front moving velocity is calculated for each time step based on the TO method. Second, we determine the total amount of water that is ahead of the advective wetting front due to diffusive effects. This amount of water is determined by integrating water content ahead of wetting front at each time step, which can be performed analytically. Finally, the water amount due to the diffusive effects is re-distributed among bins to simulate the effects of diffusion on the water content profiles.

Based on the above concepts, the developed solution in this study only involves algebraic calculations with no need to solve differential equations in determining the diffusive effects. The advective portion of the solutions is determined by solving a simple ordinary differential equation similar to the TO method. Since both diffusivity and hydraulic conductivity are a strong nonlinear function of water content, they are bin-dependent (i.e., water content dependent) which is the main reason that the Richards equation is difficult to solve numerically, especially in advection dominated problems. Our method separates advective and diffusive effects for each individual water content bin. We then evaluate the developed solution for problems related to two extreme ends of advection and diffusion by comparing the results in this study to those from numerical solutions of the Richards equation.

2. Methods

Since the main objective of this study is to incorporate soil water diffusivity into the TO method, we first briefly recapture the main ideas of the TO method (Talbot and Ogden, 2008; Ogden et al., 2015c). Please refer to Ogden et al. (2015c) for more details. The TO method is an approximate general solution method of one-dimensional unsaturated zone flow problem for infiltration, falling slugs, and vadose zone response to water table dynamics that includes gravity and capillary gradients, but neglects soil water diffusivity. The main concept is represented in Fig. 1, which illustrates that the porous medium is divided into N discrete segments of water content space called “bins”, $\Delta\theta$, between the residual water content θ_r and saturated water content, θ_s. The TO method mainly deals with three processes. The first process is the infiltration front, as shown in green in Fig. 1. The second is the falling slug as shown in purple in Fig. 1. The third is the water held up by capillarity that is in contact with a groundwater table as shown in blue in Fig. 1. In this study, our focus is on the infiltration front.

Water advances through a front that spans water content ranging from the wettest (right most) bin with the water content θ_0 to driest (left most) bin with the water content of θ_{ini} in the water content profile. The TO method is built on the same principles in deriving the Richards equation but uses the cyclic chain rule to determine how each water content bin moves in the z direction (Fig. 1) over time. The following equation of determining how wetting front z in representing the water content bin θ can be then derived,

$$\left(\frac{dz}{dt}\right)_\theta = \frac{\partial K(\theta)}{\partial\theta}\left(1-\frac{\partial\psi(\theta)}{\partial z}\right) - \frac{K(\theta)}{(\partial\theta/\partial z)}\frac{\partial^2\psi(\theta)}{\partial z^2} \tag{1}$$

where $K(\theta)$ is the hydraulic conductivity at water content θ, and ψ is the capillary pressure head. The fundamental assumption of the TO method is that the infiltration wetting front ψ is single valued everywhere along the wetting front. Ogden et al. (2015c) demonstrated that the second term in the right-hand side of Eq. (1) is small, which may be neglected. We then have the following equation,

$$\left(\frac{dz}{dt}\right)_\theta = \frac{\partial K(\theta)}{\partial\theta}\left(1-\frac{\partial\psi(\theta)}{\partial z}\right) \tag{2}$$

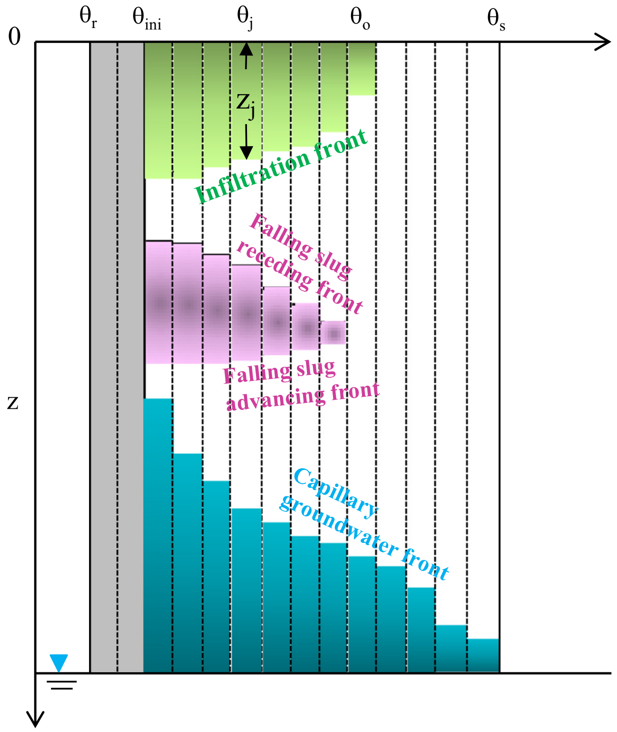

Fig. 1. Discretization in water content domain and three water content profile fronts as conceptualized in Ogden et al. (2015c) including infiltration fronts (top, green), falling slugs (middle, purple), and capillary groundwater fronts (bottom, blue) (modified based on Ogden et al. (2015c)). In this study, we mainly seek to improve infiltration fronts, where z_j varies with both water content bin θ_j and time t_i. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)