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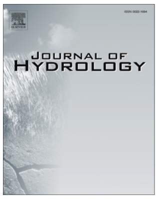

The soil water characteristic as new class of closed-form parametric expressions for the flow duration curve

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SUMMARY

The flow duration curve is a signature catchment characteristic that depicts graphically the relationship between the exceedance probability of streamflow and its magnitude. This curve is relatively easy to create and interpret, and is used widely for hydrologic analysis, water quality management, and the design of hydroelectric power plants (among others). Several mathematical expressions have been proposed to mimic the FDC. Yet, these efforts have not been particularly successful, in large part because available functions are not flexible enough to portray accurately the functional shape of the FDC for a large range of catchments and contrasting hydrologic behaviors. Here, we extend the work of Vrugt and Sadegh (2013) and introduce several commonly used models of the soil water characteristic as new class of closed-form parametric expressions for the flow duration curve. These soil water retention functions are relatively simple to use, contain between two to three parameters, and mimic closely the empirical FDCs of 430 catchments of the MOPEX data set. We then relate the calibrated parameter values of these models to physical and climatological characteristics of the watershed using multivariate linear regression analysis, and evaluate the regionalization potential of our proposed models against those of the literature. If quality of fit is of main importance then the 3-parameter van Genuchten model is preferred, whereas the 2-parameter lognormal, 3-parameter GEV and generalized Pareto models show greater promise for regionalization.

1. Introduction

The flow duration curve (FDC) is a widely used characteristic signature of a watershed, and is one of the three most commonly used graphical methods in hydrologic studies, along with the mass curve and the hydrograph (Foster, 1934). The FDC relates the exceedance probability (frequency) of streamflow to its magnitude, and characterizes both the flow regime and the streamflow variability of a watershed. It is closely related to the "survival" function in statistics (Vogel and Fennessey, 1994), and is interpreted as a complement to the streamflow cumulative distribution function (CDF). The FDC is frequently used to predict the distribution of streamflow for water resources planning purposes, to simplify analysis of water resources problems, and to communicate watershed behavior to those who lack in-depth hydrologic knowledge. One should be particularly careful to rely solely on the FDC as main descriptor of catchment behavior (Vogel and Fennessey, 1995; Westerberg et al., 2014) as the curve represents the rainfall-runoff as disaggregated in the time domain and hence lacks temporal structure (Searcy, 1959; Vogel and Fennessey, 1994).

The first application of the FDC dates back to 1880 and appears in the work by Clemens Herschel (Foster, 1934). Ever since, the FDC has been used in many fields of study including (among others) the design and operation of hydropower plants (Singh et al., 2001; Niadas and Mentzelopoulos, 2008), flow diversion and irrigation planning (Chow, 1964; Warnick, 1984; Pitman, 1993; Mallory and McKenzie, 1993), streamflow assessment and prediction (Tharme, 2003), sedimentation (Vogel and Fennessey, 1995), water quality management (Mitchell, 1957; Searcy, 1959; Jehng-Jung and Bau, 1996; Moftakhari et al., 2015), waste-water treatment design (Male and Ogawa, 1984), and low-flow analysis (Wilby et al., 1994; Smakhtin, 2001; Pfannerstill et al., 2014). Recent studies have used the FDC as a benchmark for quality control (Cole et al., 2003),

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and signature or metric for model calibration and evaluation (Refsgaard and Knudsen, 1996; Yu and Yang, 2000; Wagener and Wheater, 2006; Son and Sivapalan, 2007; Yadav et al., 2007; Yilmaz et al., 2008; Zhang et al., 2008; Blazkova and Beven, 2009; Westerberg et al., 2011; Vrugt and Sadegh, 2013; Pfannerstill et al., 2014; Sadegh and Vrugt, 2014; Sadegh et al., 2015). For instance, Vrugt and Sadegh (2013) used the fitting coefficients of a simple parametric expression of the FDC as summary statistics in diagnostic model calibration and evaluation using approximate Bayesian computation (ABC). This ABC diagnostics methodology has been introduced and described by Vrugt and Sadegh (2013) and interested readers are referred to this and subsequent publications by Sadegh and Vrugt (2014), Sadegh et al. (2015), Vrugt (2016) for further details.

Application of FDCs for hypothesis testing (Kavetski et al., 2011) can improve identifiability and help attenuate the problems associated with traditional residual-based objective (likelihood) functions (e.g. Nash–Sutcliffe, sum of squared residuals, absolute error, relative error) that emphasize fitting specific parts of the hydrograph, such as high or low flows (Schaefli and Gupta, 2007; Kavetski et al., 2011; Westerberg et al., 2011), and thereby lose important information regarding the structural inadequacies of the model (Gupta et al., 2008, 2012; Vrugt and Sadegh, 2013). The FDC is a signature watershed characteristic that along with other hydrologic metrics, can help shed lights on epistemic (model structural) errors (Euser et al., 2013; Vrugt and Sadegh, 2013). For example, Son and Sivapalan (2007) used the FDC to highlight the reasons of model malfunctioning and to propose improvements to the structure of their conceptual water balance model for the watershed under investigation. Indeed, a deep groundwater flux was required to simulate adequately dominant low flows of the hydrograph. Yilmaz et al. (2008) in a similar effort to improve simulation of the vertical distribution of soil moisture in the HL-DHM model, used the slope of the FDC as benchmark for model performance. The FDC was deemed suitable for this purpose due to its strong dependence on the simulated soil moisture distribution, and relative lack of sensitivity to rainfall data and timing errors. However, the proposed refinements of the HL-DHM model were found inadequate and this failure was attributed to the inherent weaknesses of the conceptual structure of HL-DHM.

The usefulness of duration curves (e.g. precipitation (Yokoo and Sivapalan, 2011), baseflow (Kunkle, 1962) and streamflow (flow) (Hughes and Smakhtin, 1996)) depends in large part on the temporal resolution of the data (e.g. quarterly, hourly, daily, weekly, and monthly) these curves are constructed from. FDCs derived from daily streamflow data are commonly considered to warrant an adequate analysis of the hydrologic response of a watershed (Vogel and Fennessey, 1994; Smakhtin, 2001; Wagener and Wheater, 2006; Zhao et al., 2012). For example, a FDC with a steep mid section (also referred to as slope) is characteristic for a watershed that responds quickly to rainfall, and thus has a small storage capacity and large ratio of direct runoff to baseflow. A more moderate slope, on the contrary, is indicative of a basin whose streamflow response reacts much slower to precipitation forcing with discharge that is made up in large part of baseflow (Yilmaz et al., 2008).

The shape of the FDC is determined by several factors including (amongst others) topography, physiography, climate, vegetation cover, land use, and storage capacity (Singh, 1971; Lane et al., 2005; Zhao et al., 2012; Brown et al., 2013), and can be used to perform regional analysis (Wagener and Wheater, 2006; Masih et al., 2010) or to cluster catchments into relatively homogeneous groups that exhibit a relatively similar hydrologic behavior (Sawicz et al., 2011; Coopersmith et al., 2012). Different studies have appeared in the hydrologic literature that have analyzed how the shape of the FDC is affected by physiographic factors and/or vegetation cover. Despite this progress made, interpretation of the FDC can be controversial if an insufficiently long streamflow data record is used (Vogel and Fennessey, 1994). The lower end of the FDC (low flows) is particularly sensitive to the period of study, and to whether the streamflow data includes severe droughts or not (Castellarin et al., 2004a). If the available data is sparse and does not warrant an accurate description of the FDC, then the use of an annual duration curve is advocated (Searcy, 1959; Vogel and Fennessey, 1994; Castellarin et al., 2004a,b). This curve describes the relationship between the magnitude and frequency of the streamflow for a "typical hypothetical year" (Vogel and Fennessey, 1994). To construct an annual FDC, the available data is divided into z years and individual FDCs are constructed for each year. Then, for each exceedance probability a median streamflow is derived from these z different FDCs and used to create the annual FDC. Vogel and Fennessey (1994) used this concept to associate confidence and recurrence intervals to FDCs in a nonparametric framework. One should note that the FDC of the total data record is, in general, more accurate than the annual FDC (Leboutillier and Waylen, 1993). What is more, recent studies have provided physically-based approaches, especially for tidal rivers, to extend river discharge records beyond the period of observation (Moftakhari et al., 2013; Moftakhari, 2015). Such approaches can be helpful to derive the FDC for sites with scarce or no discharge observations.

To better analyze and understand the physical controls on the FDC, it is common practice to divide the total FDC (TFDC) into a slow (SFDC) and fast (FFDC) flow component (Yokoo and Sivapalan, 2011; Cheng et al., 2012; Coopersmith et al., 2012; Yaeger et al., 2012; Ye et al., 2012). For example, Yokoo and Sivapalan (2011) concluded from numerical simulations with a simple water balance model that the FFDC is controlled mainly by precipitation events and timing, whereas the SFDC is most sensitive to the storage capacity of the watershed and its baseflow response. This type of analysis is of particular value in regionalization studies, and prediction in ungauged basins. Indeed, much effort has gone towards prediction of the FDC in ungauged basins using measurements of the rainfall-runoff response from hydrologically similar, and preferably geographically nearby, gauged basins (Holmes et al., 2002; Sivapalan et al., 2003).

In this context, one approach has been to cluster catchments into classes with similar physiographic and climatic characteristics, and then to estimate dimensionless (non-parametric) FDCs for gauged basins which in turn are then applied to ungauged basins (Niadas, 2005; Ganora et al., 2009). One such example is the work of Pugliese et al. (2014) who applied top-kriging to predict the empirical FDC in ungauged catchments. The FDCs are normalized by an index value (e.g. mean annual runoff) to generate dimensionless curves (Ganora et al., 2009; Shamseldin, 2014). A detailed review on methods for clustering of homogeneous catchments appears in Sauquet and Catalogne (2011) and Booker and Snelder (2012), and interested readers are referred to these publications for more information. Another approach has been to mimic the empirical (observed) FDC with a mathematical/probabilistic model and to correlate the fitting coefficients of such parametric expressions to physical and climatological characteristics of the watershed using regression techniques, index models, artificial intelligence, and spacial interpolation schemes (Fennessey and Vogel, 1990; Yu and Yang, 1996; Yu et al., 2002; Croker et al., 2003; Castellarin et al., 2004a,b, 2007; Li et al., 2010; Sauquet and Catalogne, 2011; Viola et al., 2011; Longobardi and Villani, 2013; Pumo et al., 2013; Mendicino and Senatore, 2013; Shamseldin, 2014; Waseem et al., 2015). Such pedotransfer functions can then be used to predict the FDC of ungauged basins from simple catchment data (e.g. soil texture, topography, vegetation cover, etc.).

Models that emulate the FDC can be grouped in two main classes: 1. Physical models that use physiographic and climatic characteristics of the watersheds (e.g. drainage area, mean areal