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Investigating relationships between rainfall and karst-spring discharge by higher-order partial correlation functions

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SUMMARY

Time series of rainfall and karst-spring discharge are influenced by various space–time-variant processes involved in the transfer of water in hydrological cycle. The effects of these processes can be exhibited in auto-correlation and cross-correlation functions. Consequently, ambiguities with respect to the effects encoded in the correlation functions exist. To solve this problem, a new statistical method for investigating relationships between rainfall and karst-spring discharge is proposed. The method is based on the determination and analysis of higher-order partial correlation functions and their spectral representations. The study area is the catchment of the Jadro Spring in Croatia. The analyzed daily time series are the air temperature, relative humidity, spring discharge, and rainfall at seven rain-gauges over a period of 19 years, from 1995 to 2013. The application results show that the effects of spatial and temporal variations of hydrological time series and the space–time-variant behaviours of the karst system can be separated from the correlation functions. Specifically, the effect of evapotranspiration can be separated to obtain the forms of correlation functions that represent the hydrogeological characteristics of the karst system. Using the proposed method, it is also possible to separate the effects of the process of groundwater recharge that occurs in neighbouring parts of a catchment to identify the specific contribution of each part of the catchment to the karst-spring discharge. The main quantitative results obtained for the Jadro Spring show that the quick-flow duration is 14 days, the intermediate-flow duration is 80 days, and the pure base flow starts after 80 days. The base flow consists of an inter-catchment groundwater flow. The system memory of the spring is 80 days. The presented results indicate the far-reaching applicability of the proposed method in the analyses of relationships between rainfall and karst-spring discharge; *e.g.*, the method can be used for (1) the identification of effects of various time series on the quick-flow, intermediate-flow and base-flow components of the discharge, (2) the detection of the seasonal effect of inter-catchment groundwater flows in the discharge, and (3) the estimation of the karst system memory. Generally, the presented approach can be applied to the qualitative analyses of the relationships between two time series whenever appropriate control time series are available.

1. Introduction

The hydrogeological characteristics of karst are complex. The structures of pores, fissures, fractures and conduits of various sizes and forms create complex conditions for groundwater flow (*e.g.*, White, 2002). Vertical representations of karst are usually distinguished into four zones: (1) soil cover, (2) the epikarst or subcutaneous zone, (3) the vadose or unsaturated zone and (4) the phreatic or saturated zone. The soil cover and the epikarst zone play an important role in the generation of overland flow and underground lateral flow, which precede the process of the vertical sinking of water through the vadose zone. In bare karst areas (karst landscape lacking soil cover), overland flow practically does not exist. The exceptions are karst fields or poljes – flat-floored depressions in karst limestone with superficial deposits that tend to accumulate water on the floor (*e.g.*, Milanović, 1981; Krešić and Stevanović, 2010). Drainage may be by either surface watercourses (when the polje is opened) or ponors (when the polje is closed). Ponors are holes or openings in the bottom of a depression where a surface stream flows either partially or completely underground into the karst groundwater system.

The process of groundwater transmission through the vadose zone can be divided into three components: (1) shaft flow through the natural and predominantly vertical shafts as a thin film of water flowing along their walls, (2) vadose flow through the enlarged joints and fractures of the vadose zone with a predominantly vertical

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direction, and (3) vadose seepage or very slow sinking through the smallest karst joints and fissures. These three components represent the input mechanism to the phreatic zone, which is characterized primarily as the area of horizontal water circulation towards the network of underground channels that transport water finally to a karst spring. Shaft flow is represented in a karst-spring hydrograph as the fast response portion, which is termed quick flow (Atkinson, 1977). Vadose seepage is represented by the slow response portion of the hydrograph, which is termed base flow. Vadose flow can be sometimes identified in a spring hydrograph as the intermediate flow located between the quick flow and the base flow. In addition to these three components, the groundwater exchange between adjacent catchments through underground piracy routes and the resulting inter-catchment groundwater flow component can be present and additionally complicate the hydrological situation (*e.g.*, Perrin et al., 2003; Le Moine et al., 2007, 2008; Jukić and Denić-Jukić, 2009; Mayaud et al., 2014).

In preliminary analyses, characteristics of underground hydrographic networks such as the degree of karstification, retention capability and groundwater hydrodynamics have been investigated by means of correlation and spectral analysis of time series of rainfall, discharge and the groundwater level (Mangin, 1984; Padilla and Pulido-Bosch, 1995; Angelini, 1997; Larocque et al., 1998; Labat et al., 2000; Mathevet et al., 2004; Rahnemaei et al., 2005; Panagopoulos and Lambrakis, 2006). Correlation and spectral analysis have found several other applications in karst hydrology. For example, it has been used to investigate hydrological processes on karst surface (Jukić and Denić-Jukić, 2008; Jemcov and Petrič, 2009), the transport properties and turbidity dynamics of karst aquifers (Bouchaou et al., 2002; Massei et al., 2006), interactions between rivers and karst aquifers during flood events (Bailly-Comte et al., 2008), characteristics of cavern conduit systems (Tam et al., 2004), hydrogeological connections in karst networks (Gill et al., 2013), storm behaviour (Herman et al., 2009), residence and response times (Bailly-Comte et al., 2011; Delbart et al., 2014), and transient inter-catchment flows (Mayaud et al., 2014). Detailed theoretical considerations about correlation and spectral analysis can be found in Jenkins and Watts (1968), Shumway and Stoffer (2000) and Box et al. (2008).

The correlation coefficient is a measure of the strength of the linear relationship between two time series. However, a correlation between two time series does not necessarily imply that one causes the other. Causation implies correlation, but correlation does not imply causation (*e.g.*, Liang, 2014); other reasons for the correlation may exist. For example, an observed correlation between two hydrological time series may not be the result of one series physically depending on the other. It may mean that another hydrological time series could control both series. Consequently, there may be ambiguities with respect to the effects of various time series encoded in the correlation coefficient that represent a significant limitation to practical application. The ambiguities may be solved using the partial correlation coefficient. The partial correlation coefficient measures the strength of the linear relationship between two time series depending on the linear effect of other (control) time series. A control time series is one that is used to extract the variance it explains from each of the two initial time series that are correlated (*e.g.*, Garson, 2012). The resulting partial correlation is thus the correlation that remains between the two initial time series once the variance explained by the control time series has been removed from each of them. The first-order partial correlation coefficient measures the effect of a single control time series, whereas higher-order partial correlation coefficients measure the cumulative effect of several control time series, where the number of control time series defines the order of partial correlation. The first- and higher-order partial correlation coefficients have been applied in hydrology and water resources research to investigate relationships between various time series (*e.g.*, Bellin and Rinaldo, 1995; Vervier et al., 1999; Treble et al., 2005; Zhang et al., 2007; Burn, 2008; Boucher et al., 2009; He et al., 2011; Vandenbohede et al., 2011; Fan et al., 2013).

The first-order partial correlation coefficients have also been calculated for the time-lagged versions of a single control time series to investigate the delayed effects of this series on the relationship between the two initial time series. The result of this calculation is a discrete function that is termed the partial correlation function. This function has two arguments: two time lags that exist between the three analyzed time series. The values of this function can be represented in the form of a matrix, which is termed the partial correlation matrix. The partial correlation matrix has been used in various fields of science (*e.g.*, Stark et al., 2006; Tkach et al., 2007). Jukić and Denić-Jukić (2011) were the first to apply the partial correlation matrix to the field of karst hydrology. To retrieve information from the coefficients in the matrix, the effect of the control time series was compared with the effect of white noise in the frequency domain; i.e., the spectral representation of partial correlation matrix was analyzed. The matrix was used with an aim to recognize the hydrological processes affecting the karst system response, determine the contribution and relative importance of each process, identify antecedent and subsequent influences of each process, and estimate the relative importance of observed and unobserved processes.

Considering higher-order partial correlation functions, a specific form is the partial autocorrelation function, which is used to identify the order of autoregressive processes (*e.g.*, Shumway and Stoffer, 2000; Box et al., 2008). This function gives the partial correlation of a time series with its own lagged values, controlling for the values of the time series at all shorter lags. It has often been applied in hydrological time series analyses, modelling and forecasting (*e.g.*, Gemitzi and Stefanopoulos, 2011; Wu and Chau, 2011; Lohani et al., 2012; Valipour et al., 2012, 2013; Belayneh et al., 2014; Cui and Singh, 2015). Other, more complex forms of higher-order partial correlation functions have not been applied in hydrology, likely because it is often difficult to find appropriate control time series and to assign a physical meaning to the values of functions.

This paper presents a new statistical method to separate the influences of various space–time-variant processes on the relationship between rainfall and karst-spring discharge. The method is based on the determination and analysis of higher-order partial correlation functions. Generally, the number of arguments of a higher-order partial correlation function is equal to the order of the function plus one. The arguments are the time lags between the analyzed time series. However, the functions introduced in this paper have only one argument, a time lag that is equivalent to the time lag of the correlation function between the two initial time series. This characteristic is crucial from the standpoint of the presentation and interpretation of the results. The most important reason to use the proposed simplified forms of higher-order partial correlation functions is that they overcome the limitation of correlation analysis regarding ambiguities with respect to the effects of control time series encoded in the auto-correlation and cross-correlation functions.

2. Methods

2.1. Correlation analysis

Let x_t and y_t, where $t = 1, 2, \ldots, n$, represent the time series of length n. The covariance between the time series x_t and y_{t+k}, where k is a time lag between two series, is obtained by:

$$c_{xy_k} = \frac{1}{n}\sum_{t=1}^{n-k}(x_t - \bar{\mu}_x)(y_{t+k} - \bar{\mu}_y), \qquad (1)$$