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Optimal input experiment design and parameter estimation in core-scale pressure oscillation experiments

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SUMMARY

This paper considers Pressure Oscillation (PO) experiments for which we find the minimum experiment time that guarantees user-imposed parameter variance upper bounds and honours actuator limits. The parameters permeability and porosity are estimated with a classical least-squares estimation method for which an expression of the covariance matrix of the estimates is calculated. This expression is used to tackle the optimization problem. We study the Dynamic Darcy Cell experiment set-up (Heller et al., 2002) and focus on data generation using square wave actuator signals, which, as we shall prove, deliver shorter experiment times than sinusoidal ones. Parameter identification is achieved using either inlet pressure/outlet pressure measurements (Heller et al., 2002) or actuator position/outlet pressure measurements, where the latter is a novel approach. The solution to the optimization problem reveals that for both measurement methods an optimal excitation frequency, an optimal inlet volume, and an optimal outlet volume exist. We find that under the same parameter variance bounds and actuator constraints, actuator position/outlet pressure measurements result in required experiment times that are a factor fourteen smaller compared to inlet pressure/outlet pressure measurements. This result is analysed in detail and we find that the dominant effect driving this difference originates from an identifiability problem when using inlet–outlet pressure measurements for joint estimation of permeability and porosity. We illustrate our results with numerical simulations, and show excellent agreement with theoretical expectations.

1. Introduction

Two key parameters influencing fluid flow in a porous medium are permeability (i.e. inverse resistance) and porosity (i.e. storage capacity). These parameters are important to characterise fluid flow in underground water resources (Cardiff et al., 2013), contaminated water disposal in underground storages (Song and Renner, 2007), and subsurface hydrocarbon reservoirs (Wang and Knabe, 2011). Indeed, permeability and porosity estimates are used to initialise reservoir simulations, optimise the number of wells and their locations, and drilling and completion procedures.

At the core scale, estimation of both parameters *locally* may be carried out by performing an experiment on a cylindrically-shaped core sample of the porous medium, using either steady-state (SS), unsteady-state (USS), or pressure oscillation (PO) measurements. In an SS experiment a constant pressure difference is applied across the axis of the core sample and subsequently the flow rate is measured after the SS condition has been established. Permeability is then estimated based on the relationship between the flow rate and the pressure drop. In an USS experiment an impulse or step pressure change is applied at the upstream side of the sample while the pressure change is recorded downstream. The observed response is then analysed either graphically or numerically to estimate either permeability or porosity. Similarly, in a PO experiment, the recorded downstream pressure response is analyzed for parameter estimation – the difference being that an oscillatory pressure signal is applied upstream. The attenuation and phase shift between the up- and downstream signals are then translated into parameter estimates (Fischer, 1992; Heller et al., 2002). The oscillatory signal is usually a single sinusoid with a frequency and amplitude specified by the experimenter. The amplitude of

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the upstream signal is however bounded by the limits of the actuator. In cases where a rather high actuator frequency is necessary to take into account geometrical and physical properties of the sample, (Boitnott, 1997) suggested the use of input signals with complex shapes including the required high frequencies.

The consensus in the literature is that a PO experiment has several advantageous properties not shared by its SS and USS counterparts, e.g., less experiment time, less stress on the core sample, and the possibility of simultaneously estimating permeability and porosity (Bernabé and Evans, 2006; Song and Renner, 2007). The effectiveness of PO experiments for the estimation of permeability has been demonstrated in different set-ups (Heller et al., 2002; Wang and Knabe, 2011; Suri et al., 1997; Hasanov and Batzle, 2013; Boitnott, 1997). Despite its advantageous properties, however, it is observed that measurements can result in large uncertainties in the estimates, particularly for porosity (Bernabé and Evans, 2006; Song and Renner, 2007; Wang and Knabe, 2011). Porosity estimates with an uncertainty exceeding an order of magnitude, or that have negative values, have been reported (Song and Renner, 2007; Bernabé and Evans, 2006). (Negative values can however be easily circumvented by using log-transformed parameters). One cause is measurement noise, but in this paper we show that other ones also play an important role.

Furthermore, it is important to be able to reduce the experiment time without loss of accuracy. In such a case, more core samples can be analysed in a given time, which consequently reduces the experiment costs. Analogously, given a maximum experiment time, it is important to get the best possible estimates.

Clearly, the challenge of estimating permeability and porosity with high accuracy remains, especially in evaluating the production potential of tight formations in unconventional hydrocarbon reservoirs (Wang and Knabe, 2011) or the sealing characteristics of the cap rock in underground storage (Song and Renner, 2007).

Motivated by the above problems we raise the question whether we can, for a PO experiment, design the applied upstream pressure signal and utilise the degrees of freedom (DOF) in the experiment set-up in order to increase parameter accuracies. The dependence of the accuracy of the estimates on the selected driving frequency has been first pointed out in Kranz et al. (1990), although no investigation into this topic was pursued. From this question, we define the following optimization problem: find the minimal experiment time required to guarantee user-imposed variance constraints on the estimates by utilising DOF in the experiment set-up as well as designing the to-be-applied upstream pressure signal, ensuring that this signal has an amplitude that honours the actuator limits. Note that the solution can also be used to maximise the accuracy of the estimates for a given experiment length. To address this optimization problem we use techniques from Experiment Design.

Experiment Design addresses the long-standing issue of the lack of accurate parameter estimates inferred from collected data, particularly at the catchment scale. This issue is widely recognised; see for instance (Gupta and Sorooshian, 1985; Kleissen et al., 1990; Beven and Binley, 2012; Wagner, 1992; Yapo et al., 1996) and the nice review of Kool et al. (1987). Some of the earliest works (Sorooshian et al., 1983; Sorooshian and Gupta, 1983, 1985) in Experiment Design (although not recognised under this name at that time) showed that concepts such as *parameter correlation*, *identifiability*, *observability*, and experiment length strongly affect the quality of the parameter estimates (i.e. their variances). These works and those of Kuczera (1983) and Kuczera (1983) were some of the first to quantitatively evaluate parameter uncertainty within a Bayesian framework. They provided measures to find the best possible calibration data for computer models, using *a posteriori* data, i.e. data from an experiment that had already taken place. Other works (Wagner, 1992; Mahar and Datta, 2001) analysed the role of tracer observations that influence parameter identifiability, and identifiability of unknown pollution sources. The works (Hsu and Yeh, 1989; Nishikawa and Yeh, 1989; McCarthy and Yeh, 1990) were the first to consider optimal experiment design for groundwater hydrology *prior* to the actual inference experiment; they mainly searched for optimal pumping and observation wells, keeping the pumping rates constant, such that the experiment cost could be minimized subject to maximizing the overall accuracy in the parameters (using a D-optimality criterion). More recently, a Bayesian methodology (Leube et al., 2012) was developed to find the optimal investigation strategy, or sampling pattern, prior to the actual experimental campaign.

We will take a non-Bayesian approach from linear systems theory (Bombois et al., 2006) and apply it to the core-scale PO experiment introduced above. The method is different to the Bayesian methods in the sense that an optimal spectrum of the input signal is calculated prior to the actual experiment, whereas in the above methods the spectra of the inputs are not design variables. This optimal spectrum reveals e.g. the time scales that are important for accurate parameter estimation. We also consider variance constraints on the individual parameters, which is particularly important to use for systems that have low sensitivities for some parameters (in which case the D-optimality criterion, as used by e.g. (Nishikawa and Yeh, 1989), can be ill-chosen).

In this paper, we tackle the experiment design problem as follows. We perform parameter estimation using ordinary least squares using the measured noise-corrupted downstream pressure signal (Ljung, 1999; Aster et al., 2005). This signal is deduced from the governing equations and boundary conditions, and depends on the applied upstream signal. One benefit of this method is that it can deal in a rather easy manner with (coloured) measurement noise; see (Ljung, 1999) for details. A second benefit is that a frequency-domain expression of the covariance matrix of the estimates can be formulated. This expression, which we introduce in Section 3, is a function of the power spectrum of the applied signal and the DOF of the experiment set-up. Consequently, we can formulate the above optimization problem (of minimising the experiment time subject to parameter variance constraints and actuator bounds by designing the optimal input signal and DOF of the set-up) mathematically. We shall limit ourselves to sinusoidal and square-wave actuator signals. The latter is easy to generate by rapidly switching between two actuator levels, which can be done with current vibration exciters (Heller et al., 2002). Other reasons for this choice are explained in Section 5.

We apply our method to the Dynamic Darcy Cell experiment set-up, as detailed in Heller et al. (2002), but we stress that our methodology can be applied to many other set-ups as well. The DOF in the Dynamic Darcy Cell set-up are the inlet volume and outlet volume. We introduce the Dynamic Darcy Cell in Section 2 and show how to apply sinusoidal and square wave signals to the set-up. Two types of measurements are then introduced: inlet pressure/outlet pressure measurements (Direct Method) and actuator position/outlet pressure measurements (Indirect Method). The former is one of the current ways to estimate parameters (Heller et al., 2002), in particular using sinusoidal signals. The latter has, to the authors' knowledge, not been investigated before. For both cases, we focus on square wave input signals, for which we prove that shorter experiment lengths than for sinusoidal ones can be obtained. We explain the data collection and estimation procedure in Section 3, and give an expression for the covariance matrix of the parameter estimates. In Sections 4 and 5 we use this expression to compute the optimal sinusoidal and square wave signals and DOF that minimize the experiment time for the estimation of permeability and porosity for the Direct and Indirect Methods. In the absence of a physical set-up, we illustrate the experiment design results by simulating the noise-corrupted physical system and