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Three dimensional analysis of unconfined seepage in earth dams by the weak form quadrature element method

Shuai Yuan [a,*], Hongzhi Zhong [b]

[a] Institute of Geotechnical Engineering, School of Highway, Chang'an University, Xi'an 710064, PR China
[b] Department of Civil Engineering, Tsinghua University, Beijing 100084, PR China

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SUMMARY

It remains challenging to determine the unknown free surface in three dimensional unconfined seepage in earth dams. A number of iterations are frequently required which make the problem computationally expensive. In the present research, a weak form quadrature element formulation is presented for three dimensional analysis of unconfined seepage which is an extension of the recently established method for two dimensional seepage problems. "Free points" are introduced by the interpolation of which the free surfaces are smoothly approximated. Grid lines are constructed in the element and the "free points" are confined to the lines when updated. An interpolatory scheme for locating the exit points is proposed. Formulations and procedures of the method are given in detail. Results of numerical examples are compared with available analytical solutions and numerical solutions in the literature and agreement is reached demonstrating the efficiency and reliability of the present formulation.

1. Introduction

Seepage analysis has found widespread applications in the fields of civil engineering, oil engineering, environmental engineering, etc. The most difficult step in unconfined seepage problems is to find the location of the free surface which is unknown before computation and must be determined by iterative process. Closed form solutions are hardly available for the complexity of practical problems. So far, numerical methods have been widely applied to deal with unconfined seepage problems such as the finite element method (Bathe and Khoshgoftaar, 1979; Chung and Kikuchi, 1987; Lacy and Prevost, 1987; Gioda and Gentile, 1987; Ahmed and Bazaraa, 2009; López-Querol et al., 2011; Kazemzadeh-Parsi and Daneshmand, 2012, 2013), the boundary element method (Chang, 1988; Rizos and Karabalis, 1992), the finite difference method (Lee and Leap, 1997; Koo and Leap, 1998), and the finite volume method (Darbandi et al., 2007; Bresciani et al., 2012).

Those numerical methods can mainly be put into two categories, namely, adaptive mesh algorithms (Chung and Kikuchi, 1987; Fenton and Griffiths, 1997; Darbandi et al., 2007; Ouria and Toufigh, 2009; Shahrokhabadi and Toufigh, 2013), and fixed mesh algorithms (Bathe and Khoshgoftaar, 1979; López-Querol et al., 2011; Kazemzadeh-Parsi and Daneshmand, 2012). In adaptive mesh approaches, computation is based on the domain below the free surface that varies during the iterative process. In fixed mesh techniques, however, the whole domain is considered and the free surface is dealt with by setting different soil properties for the parts below and above the free surface. Essentially, the geometrical nonlinearity involved in the problem is transformed into material nonlinearity in fixed mesh methods. Generally, high accuracy is achieved by the first type as the real field is considered and the meshes conform to the free surface. However, requirement of computational resource is made large due to regeneration of meshes. Mesh distortion may occur in the neighborhood of the free surface leading to high numerical errors to appear. Relatively, the fixed mesh methods are stable and can be applied to coupled analysis of flow and deformation in the whole solution domain. Nevertheless, most fixed mesh methods are inflicted by the requirement of advanced theories which are unfamiliar to engineers (Bresciani et al., 2012), and low accuracy is obtained.

The weak form quadrature element method (QEM) is a high order numerical algorithm with rapid convergence as compared with the finite element method (Zhong and Yu, 2007; Mo et al., 2009; Zhong and Gao, 2010; Zhong and Wang, 2010; He and

* Corresponding author. Tel.: +86 15209269763.
E-mail address: yuan_shuai@126.com (S. Yuan).

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Zhong, 2012). In the QEM, large elements are used and accuracy can be improved by increase of the approximation order. The method has recently been applied to two dimensional unconfined seepage in earth dams with very simple geometric shapes (Yuan and Zhong, 2015). In the present research, a three dimensional weak form quadrature element formulation using adaptive mesh is presented for unconfined seepage in earth dams with arbitrary shapes. "Free points" are introduced by the interpolation of which the free surfaces are smoothly approximated. Grid lines similar to those "vectors" in Kazemzadeh-Parsi and Daneshmand (2012) and "straight lines" in Fenton and Griffiths (1997) are constructed and the "free points" are confined to the lines when updated. Based on the element, grid lines in the present formulation are widely applicable as compared with those in the literature. An interpolatory scheme for locating the exit points is proposed. By using large elements, mesh distortion can be alleviated and the numerical implementation is straightforward and simple. Formulations and procedures are given in detail. Results of numerical examples are compared with available analytical solutions and numerical solutions in the literature and agreement is reached demonstrating the efficiency and reliability of the present formulation.

2. Formulation

2.1. Problem statement

The differential equation for steady seepage is the combination of Darcy's law and continuity of pore fluid:

$$\mathrm{div}(\mathbf{K}\nabla h) = 0 \tag{1}$$

where $\mathbf{K}$ is the permeability matrix. h, the total head or piezometric head, is defined as

$$h = z + \frac{p}{\rho g} \tag{2}$$

where z is the elevation head above a chosen datum, p is the pore pressure, ρ is the density of the liquid, and g is the acceleration due to gravity. For isotropic soils the equation becomes

$$\nabla^2 h = 0 \tag{3}$$

A typical three dimensional seepage problem with a free surface is shown in Fig. 1.

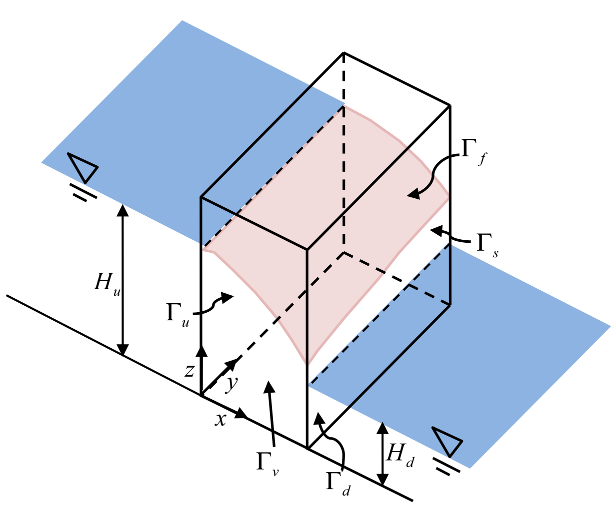

Fig. 1. Three dimensional unconfined seepage.

Generally, boundary conditions of the problem can be classified into four types (see Fig. 1):

A. Prescribed total head boundary conditions on the upstream and downstream surfaces

$$h = H_u \quad \text{on} \quad \Gamma_u \tag{4}$$

$$h = H_d \quad \text{on} \quad \Gamma_d \tag{5}$$

B. Prescribed flux boundary conditions

$$v_n = 0 \quad \text{on} \quad \Gamma_v \tag{6}$$

C. Seepage surface

$$h = z \quad \text{on} \quad \Gamma_s \tag{7}$$

D. Free surface

$$h = z \quad \text{on} \quad \Gamma_f \tag{8}$$

$$v_n = 0 \quad \text{on} \quad \Gamma_f \tag{9}$$

where v_n is the normal velocity. Two boundary conditions are given on the free surface, only one of which is applied for solution and the other is used to update the vertical position of the "free points".

2.2. Weak form quadrature element formulation

The weak form description of Eq. (1) is obtained by the principle of virtual work as

$$\int_{A_n} \delta h v_n \mathrm{d}A + \int_V (\nabla \delta h)^T \mathbf{K} \nabla h \mathrm{d}V = 0 \tag{10}$$

where A_n is the surface with prescribed normal velocity and V is the solution domain under the free surface.

In the QEM, the problem domain is first discretized into a few subdomains (elements) where numerical integration can be carried out. Then every subdomain is transformed onto the standard computational domain, i.e.

$$\begin{cases} x = x(\xi, \eta, \varsigma) \\ y = y(\xi, \eta, \varsigma) \quad -1 \leqslant \xi, \eta, \varsigma \leqslant 1 \\ z = z(\xi, \eta, \varsigma) \end{cases} \tag{11}$$

where x, y and z are coordinates in physical domain; ξ, η and ς are coordinates in the standard domain. With the chain rule of differentiation

$$\begin{Bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial \varsigma} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \varsigma} & \frac{\partial y}{\partial \varsigma} & \frac{\partial z}{\partial \varsigma} \end{bmatrix} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} = \mathbf{J} \begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} \tag{12}$$

and

$$\begin{Bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{Bmatrix} = \mathbf{J}^{-1} \begin{Bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial \varsigma} \end{Bmatrix} \tag{13}$$

where $\mathbf{J}$ is the Jacobian matrix. Introduction of Lobatto quadrature into Eq. (10) yields

$$\sum_{i=1}^{N_\xi} \sum_{j=1}^{N_\eta} \sum_{k=1}^{N_\varsigma} W_i W_j W_k |\mathbf{J}|_{ijk} (\nabla \delta h)_{ijk}^T \mathbf{K}_{ijk} (\nabla h)_{ijk} + \sum_{m=1}^{N_V} \sum_{i=1}^{N_{m1}} \sum_{j=1}^{N_{m2}} W_i W_j |\bar{\mathbf{J}}|_{ij} \delta h_{ij} v_{nij} = 0 \tag{14}$$