Journal of Hydrology 533 (2016) 412–420

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

Technical Note

The effects of oscillation period on groundwater wave dispersion in a sandy unconfined aquifer: Sand flume experiments and modelling

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ARTICLE INFO

Article history:
Received 25 September 2015
Received in revised form 15 December 2015
Accepted 17 December 2015
Available online 21 December 2015
This manuscript was handled by Corrado Corradini, Editor-in-Chief, with the assistance of Adrian Deane Werner, Associate Editor

Keywords:
Groundwater wave
Hysteresis
Unsaturated flow
Seepage face
Richards' equation

SUMMARY

This paper presents a new laboratory sand flume dataset on the propagation of groundwater waves in an unconfined sandy aquifer with a vertical boundary subject to simple harmonic forcing with a wide range of oscillation period from 10.7 s to 909 s. The data is unique in that it covers a much wider range of non-dimensional aquifer depths, $n\omega d/K$ (where n is the porosity, ω is the angular frequency, d is the aquifer depth and K is the hydraulic conductivity) than has been previously investigated. Both the amplitude decay rate and rate of increase in phase lag of the water table waves are observed to monotonically increase with increasing oscillation frequency (increasing $n\omega d/K$). This is in contrast to existing theoretical dispersion relations which predict: (1) zero phase lag or standing wave behaviour and (2) an asymptotic decay rate as the frequency increases. Possible influences on the experimental data including sand packing, measurement location, finite amplitude wave effects, unsaturated zone truncation and multiple wave mode effects are unable to explain the discrepancy. The data was also compared against numerical solutions of *Richards*' equation with and without hysteresis and in both cases, the same qualitative behaviour as the analytic solutions described above is found. The discrepancy between data and predictions remains unexplained and highlights a knowledge gap that requires further investigation. These findings relate directly to practical applications in the field of surface–groundwater interactions such as the influence of wave forcing of coastal aquifers on contaminant transport, sediment mobility and salt-water intrusion all of which are influenced by the dispersion of the groundwater wave.

1. Introduction

Coastal aquifers around the world are exploited by a range of uses including agriculture, potable water supply and waste water disposal and these aquifers are subject to the influence of groundwater waves induced by oceanic oscillations (waves and tides). The propagation of groundwater waves has been shown to have important implications for the mixing of oceanic and sub-surface water masses at the coastal margin (e.g. Li et al., 1999; Robinson et al., 2006; Xin et al., 2010) and also the mobility of sediments on beaches (e.g. Elfrink and Baldock, 2002; Xin et al., 2010; Bakhtyar et al., 2011). In particular, the speed of propagation and decay of the water table wave will dictate the magnitude and variation in hydraulic gradients near the boundary which in turn control flow rates and thus the extent of mixing processes such as salt-water intrusion and contaminant transport.

The dispersion of groundwater waves has received theoretical attention in the literature including the influence of non-hydrostatic pressure and capillarity. The simplest case is that of simple harmonic forcing of an unconfined aquifer across a vertical interface,

$$h_o = d + A\cos(\omega t) \tag{1}$$

where h_o is the driving head [L], d is the mean driving head [L], A is the driving head amplitude [L] and $\omega = 2\pi/T$ [T^{-1}] is the oscillation frequency and T is the oscillation period [T]. Under the assumption of small-amplitude oscillations ($A \ll d$), the form of the water table wave in response to this forcing is (e.g. Steggewentz, 1933; Parlange et al., 1984; Nielsen, 1990; Barry et al., 1996; Li et al., 2000b),

$$\eta(x,t) = ARe\{e^{-kx}e^{i\omega t}\} = ARe\{e^{-(k_r+ik_i)x}e^{i\omega t}\} = Ae^{-k_r x}\cos(\omega t - k_i x) \tag{2}$$

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http://dx.doi.org/10.1016/j.jhydrol.2015.12.032
0022-1694/

where η is the water table elevation [L] relative to the mean water level and $k = k_r + ik_i$ is the water table wave number which describes the dispersive properties of the wave where k_r is the decay rate [L^{-1}] of the water table wave amplitude and k_i is the rate of increase in phase lag [L^{-1}] of the water table wave with increasing distance landward and $i = \sqrt{-1}$.

Existing wave numbers derived from experimental (sand flume and Hele-Shaw cell) and field observations have been limited to non-dimensional aquifer depths $n\omega d/K < 41$ where n is the specific yield [–] and K is the saturated hydraulic conductivity [LT^{-1}]. To put these values into context, a groundwater wave induced by a semi-diurnal tide in a 10 m deep sandy beach aquifer ($T = 12.25$ h; $n = 0.3$; $K = 4 \times 10^{-4}$ m/s) corresponds to $n\omega d/K = 1.1$ whereas a 10 s wave forcing the same aquifer corresponds to $n\omega d/K = 4700$. That is, existing theories on water table wave dispersion are yet to be tested against data on the propagation of high frequency groundwater waves (large $n\omega d/K$). This paper addresses this gap in knowledge and presents a comprehensive new database of wave numbers derived from controlled sand flume experiments with an experimental parameter range of $4 < n\omega d/K < 415$.

2. Existing analytical dispersion relations

The following sections summarise the theoretical development in the literature that has led to a range of water table wave dispersion relations based on the consideration of different physical influences such as vertical flows (non-hydrostatic pressure) and capillarity. All are based on the assumption of small amplitude waves propagating in a homogeneous, isotropic aquifer.

2.1. Shallow, capillarity free aquifer

The simplest theory stems from the assumptions of a shallow (i.e. hydrostatic pressure), capillarity free aquifer which leads to the "Bousinessq" wave number valid for $n\omega d/K \ll 1$ (e.g. Todd, 1959),

$$kd = \sqrt{i\frac{n\omega d}{K}} \tag{3}$$

That is, the shallow aquifer theory predicts the rate of decay to be equal to the rate of increase in phase lag ($k_r = k_i$). This is in clear contrast to available field and laboratory data which indicates that $k_r \neq k_i$ (e.g. Nielsen, 1990; Aseervatham, 1994; Kang, 1995; Raubenheimer et al., 1999; Cartwright et al., 2003, 2004; Cartwright, 2004).

2.2. Shallow aquifer with capillarity effects

Barry et al. (1996) followed the approach of Parlange and Brutsaert (1987) and applied the non-hysteric Green and Ampt (1911) model of the capillary fringe to correct the shallow aquifer theory for capillarity effects and found,

$$k_r = \sqrt{\frac{n\omega}{2d}\left[\frac{1}{\sqrt{K^2 + (\omega n H_\psi)^2}} + \frac{\omega n H_\psi}{K^2 + (\omega n H_\psi)^2}\right]} \tag{4}$$

$$k_i = \sqrt{\frac{n\omega}{2d}\left[\frac{1}{\sqrt{K^2 + (\omega n H_\psi)^2}} - \frac{\omega n H_\psi}{K^2 + (\omega n H_\psi)^2}\right]} \tag{5}$$

where H_ψ is the equivalent saturated height of the capillary fringe [L] which is found by integrating the effective saturation from the water table upwards,

$$H_\psi = \int_h^\infty \frac{\theta - \theta_r}{\theta_s - \theta_r} dz \tag{6}$$

where h is the water table elevation [L], z is the elevation [L] and θ is the volumetric water content [–] and the subscripts s and r denote saturated and residual quantities respectively.

In essence, the work of Barry et al. (1996) demonstrates that the presence of moisture above the water table acts to reduce the dispersion of the water table wave (slower rates of decay and phase lag increase). In other words, the wave number k is smaller with capillarity effects than without.

2.3. Non-shallow, capillarity free aquifer

Building on from the experimental Hele-Shaw cell work of Aseervatham (1994), Nielsen et al. (1997) developed a theory quantifying the influence of vertical flow effects (non-hydrostatic pressure) on periodic groundwater flow. First a 2nd order (in $n\omega d/K$) dispersion relation was derived as,

$$kd = \sqrt{\frac{3}{2}}\sqrt{-1 + \sqrt{1 + \frac{4}{3}i\frac{n\omega d}{K}}} \tag{7}$$

which was then extended to infinite order,

$$kd \tan kd = i\frac{n\omega d}{K} \tag{8}$$

The implications of the exact solution (Eq. (8)) is that in the high frequency limit ($n\omega d/K \to \infty$): (1) the theory predicts a zero phase lag ($k_i = 0$) corresponding to a standing wave scenario and (2) the amplitude decay rate has an asymptotic value of $k_r = \pi/2d$. The present data however will be shown to contradict this with both k_r and k_i observed to monotonically increase with increasing $n\omega d/K$.

2.4. Non-shallow aquifer with capillarity effects

The non-shallow aquifer theory of Nielsen et al. (1997) (Eq. (8)) was extended to include capillarity effects via theoretical and empirical approaches as outlined in the following.

2.4.1. The non-hysteretic Green and Ampt model

Li et al. (2000a) adopted the theoretical, non-hysteretic Green and Ampt (1911) model of the capillary fringe and derived the following modified form of Eq. (8),

$$kd \tan kd = i\frac{n\omega d}{K + i\omega n H_\psi} \tag{9}$$

As per the capillarity free expression (Eq. (8)), in the high frequency limit this modified equation also predicts zero-phase lag and asymptotic decay rate.

2.4.2. The hysteretic dynamic effective porosity model

Nielsen and Perrochet (2000a,b) conducted sand column experiments which indicated that the Green and Ampt (1911) model was unable to replicate the observed relationship between the total moisture in the column and the water table fluctuations. To account for this Nielsen and Perrochet (2000a,b) introduced the concept of a dynamic effective porosity,

$$n_\omega \frac{\partial h}{\partial t} = n\frac{\partial h_{tot}}{\partial t} \tag{10}$$

where n_ω is the dynamic effective porosity [–], h is the water table elevation [L] and h_{tot} is the equivalent saturated height of moisture in the vertical = $d + H_\psi$ [L]. n_ω is complex in nature to account for the fact that fluctuations in h_{tot} are observed to be both damped ($|n_\omega|$) and lag ($-\mathrm{Arg}\{n_\omega\}$) those in the water table h (cf. Fig. 4, Nielsen and Perrochet, 2000a,b).