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Higher-order approximation of contaminant transport equation for turbulent channel flows based on centre manifolds and its numerical solution

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SUMMARY

The contaminant transport process governed by the advection–diffusion equation plays an important role in modelling industrial and environmental flows. In this article, our aim is to accurately reduce the 2-D advection–diffusion equation governing the dispersion of a contaminant in a turbulent open channel flow to its 1-D approximation. The 1-D model helps to quickly estimate the horizontal size of contaminant clouds based on the values of the model coefficients. We derive these coefficients analytically and investigate numerically the model convergence. The derivation is based on the centre manifold theory to obtain successively more accurate approximations in a consistent manner. Two types of the average velocity profile are considered: the classical logarithmic profile and the power profile. We further develop the one-dimensional integrated radial basis function network method as a numerical approach to obtain the numerical solutions to both the original 2-D equation and the approximate 1-D equations. We compare the solutions of the original models with their centre-manifold approximations at very large Reynolds numbers. The numerical results obtained from the approximate 1-D models are in good agreement with those of the original 2-D model for both the logarithmic and power velocity profiles.

1. Introduction

The Taylor dispersion model has been extensively applied to many practical problems of pollutant dispersion in environmental engineering (Ani et al., 2009; Chatwin and Allen, 1985; Chen, 2013; Wu and Chen, 2014; Zeng et al., 2014). Taylor (1953, 1954) derived a partial differential equation which governs the long-term transport of a contaminant in shear flows in a pipe. He assumed that there is a balance between the dominant processes within the flow – advection and diffusion. This balance can be explained as follows: when the contaminant is released into the flow, its concentration changes because of the velocity shear, and at the same time, it is smeared out across the flow because of the diffusion. After a long time, the contaminant cloud extends over a long distance along the pipe, in x-direction. As a result of this combined action of the advection and diffusion, the concentration variation in x-direction becomes slow. Taylor (1954) described the dynamics of this long-term evolution in terms of the depth-averaged concentration, C, by the following 1-D equation

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2}, \tag{1.1}$$

where D is the constant molecular diffusion coefficient in a laminar flow; U the mean velocity of the flow; and t the time. Aris, 1956 used a "concentration moment" method and built a new basis for the Taylor's analysis by ignoring restrictions on the concentration distribution. The Taylor's and Aris's analyses were extended by Elder (1959) to describe the longitudinal diffusion in the turbulent flow in an open channel, based on the von Karman logarithmic velocity profile. In this work, the longitudinal diffusion coefficient was deduced to be $5.9v_*h$, where v_* is the friction velocity and h the channel depth. Experimental investigations conducted by Sayre and Chang (1968) in laboratory channels revealed that the diffusion coefficient D is not actually constant and its value varies from 3 to 13. Such a wide variation is caused by the significant effect of the velocity variation in the vertical direction. Similar approximations to these values were derived by Chatwin (1970) using an asymptotic series analysis. The studies of Taylor and Aris were followed by extensive research on modelling of dispersion in

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shear flows using a variety of techniques. More details can be found in Frankel and Brenner (1989), Gill and Sankarasubramanian (1970), and Smith (1987). As can be seen from these studies, the longitudinal diffusion is quite sensitive to the vertical distribution of the velocity.

Using centre manifold theory, Mercer and Roberts (1990) derived a low-dimensional depth-averaged model as an approximation to the 2-D advection–diffusion equation governing the longitudinal dispersion of contaminants in a laminar open channel flow. They conducted an accurate modelling and analytically deduced higher-order extensions to the Taylor's Eq. (1.1). They also derived modified initial conditions that guarantee exponentially fast convergence of the 1-D and 2-D modelling. In order to increase accuracy some authors designed zonal models. Chikwendu and Ojiakor (1985) divided the channel into two zones, with a fast zone on top of a slow zone at the bottom. They assumed that the contaminant is well mixed in each zone, introduced the cross-averaged concentration in each zone and approximated the diffusion between them by Newton's law. A system of coupled equations was empirically derived for the averaged concentrations in the zones. Later on, Roberts and Strunin (2004) constructed a two-zone model of contaminant dispersion in a Poiseuille channel flow based on centre manifolds. The mentioned two-zone models were applied to laminar flows for which the diffusion coefficient was assumed to be constant. Strunin (2011) applied the centre manifold theory to turbulent flows and deduced an advection–diffusion–dispersion equation for the depth-averaged concentration for logarithmic and power velocity profiles. The classical logarithmic velocity profile has the form

$$v = \frac{v_*}{\kappa}\ln\left(\frac{v_* y}{\nu}\right) + A, \tag{1.2}$$

where ν is the kinematic molecular viscosity; κ the von Karman constant ($\kappa = 0.4$) and A an empirical constant. Taking into account the effects of the viscosity and the Reynolds number in the inertial layer, Barenblatt (1993, 2000) suggested an alternative power-like velocity profile,

$$\frac{v}{v_*} = \left(\frac{1}{\sqrt{3}}\ln \mathrm{Re} + \frac{5}{2}\right) y_+^{\frac{3}{2\ln \mathrm{Re}}}, \tag{1.3}$$

where $y_+ = v_* y/\nu$ is the dimensionless cross-flow coordinate; and Re the Reynolds number. He showed that the scaling law (1.3) gives an accurate description of the mean velocity distribution over the self-similar intermediate region adjacent to the viscous sublayer for a wide variety of boundary layer flows. He also studied the dispersion for three different configurations of sources located on the bottom using the power velocity profile (Barenblatt, 2003) and revealed how these solutions depend on the Reynolds number.

In the present work, we separately investigate the longitudinal dispersion of contaminants for the logarithmic and power velocity profiles, at large Reynolds numbers. The velocity profile is taken to be either (1.2) or (1.3) across the channel from the bottom, except for the narrow viscous sublayer near the bottom, to the surface where waves are neglected. The governing 2-D advection–diffusion equation has the form

$$\frac{\partial c}{\partial t} + v(y)\frac{\partial c}{\partial x} = \frac{\partial}{\partial y}\left[D(y)\frac{\partial c}{\partial y}\right], \tag{1.4}$$

where $v(y)$ is the velocity directed along the channel; $D(y)$ the coefficient responsible for the turbulent diffusion across the channel and $c(x, y, t)$ the contaminant concentration. Eq. (1.4) is complemented by the no-flux boundary conditions

$$D\left.\frac{\partial c}{\partial y}\right|_{y=0} = D\left.\frac{\partial c}{\partial y}\right|_{y=h} = 0. \tag{1.5}$$

An expression for the diffusion coefficient $D(y)$ is deduced from the Prandtl formula for the stress (Strunin, 2011),

$$D(y) = \frac{K v_*^2}{\partial v/\partial y}, \tag{1.6}$$

where $K = 1$. Eqs. (1.2)–(1.6) form a self-consistent dynamical system to be converted into the 1-D equation for the depth-averaged concentration $C(x, t)$,

$$\frac{\partial C}{\partial t} = g_1\frac{\partial C}{\partial x} + g_2\frac{\partial^2 C}{\partial x^2} + g_3\frac{\partial^3 C}{\partial x^3}\ldots, \tag{1.7}$$

where g_1, g_2 and g_3 are the coefficients analytically derived by Strunin (2011) for both the logarithmic and power velocity profiles. The evolution Eq. (1.7) can be used to roughly predict the spreading of the contaminants along the channel. The analytically derived coefficients g_n are responsible for the effects of advection, diffusion and dispersion. They are determined as functions of parameters characterising the flow such as the Reynolds number and the von Karman constant κ. Even without solving Eq. (1.7), one can quickly estimate the size of the contaminant cloud based on the coefficients g_n. The characteristic distances over which the substance propagates during a period of time T are (e.g., by dimensional analysis)

$$L_1 = g_1 T \quad \text{due to the advection}, \tag{1.8}$$

$$L_2 = (g_2 T)^{1/2} \quad \text{due to the diffusion}, \tag{1.9}$$

$$L_3 = (g_3 T)^{1/3} \quad \text{due to the dispersion}. \tag{1.10}$$

From (1.8)–(1.10), we have $g_2/g_1 = L_2^2/L_1$ and $g_3/g_1 = L_3^3/L_1$. Therefore, for the same advection distance L_1, the larger the ratios g_2/g_1 and g_3/g_1, the larger the effects of diffusion and dispersion. Mohammed et al. (2014) also derived the coefficient g_4 for the logarithmic velocity profile. They employed the one-dimensional integrated radial basis function network (1D-IRBFN) method as a numerical approach to demonstrate that the solution of the derived 1-D model is in good agreement with that of the original 2-D model. The 1D-IRBFN and IRBFN-based methods have been successfully verified through several engineering problems such as viscous flows (Mai-Duy and Tanner, 2007; Ngo-Cong et al., 2012), natural convection flows (Ngo-Cong et al., 2012) and structural analysis (Ngo-Cong et al., 2011).

In the present study, we derive the higher-order coefficients g_5 and g_6 for the logarithmic profile and the coefficients g_4, g_5 and g_6 for the power profile and investigate the effects of these coefficients on the model solution. Also, we further develop the 1D-IRBFN method for solving high-order partial differential equations (up to 6th-order) and use the method to solve the 2-D advection–diffusion Eq. (1.4) and the depth-averaged 1-D Eq. (1.7) for both the logarithmic and power velocity profiles.

The paper is organised as follows. Section 2 briefly describes the centre manifold approach and its application to derive Eq. (1.7) for the power and logarithmic velocity profiles. Section 3 presents the 1D-IRBFN numerical method, followed by the discussion of numerical results in Section 4. Section 5 concludes the paper.

2. Centre manifold technique for advection–diffusion in an open channel

In a shear channel flow, there are two competing factors governing the distribution of contaminants: (i) the cross-flow diffusion which tends to quickly spread the contaminant in the vertical direction and ensure smooth distribution in this direction and (ii) the velocity shear which creates non-uniformity of the concentration across the channel because particles near the surface drift faster than particles near the bottom (Mercer and Roberts, 1990). As a result of co-action of these factors, the contaminant concentration