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Technical Note

Power law breakthrough curve tailing in a fracture: The role of advection

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SUMMARY

We offer an explanation of the strongly tailed solute breakthrough curve typically observed when a tracer test is conducted in fractured bedrock. In this example, we limit the model to a single planar fracture of varying aperture. Flow heterogeneity derives from variable fracture aperture, which implies variable transmissivity (T). The analysis employs a physically based model well-suited to strong heterogeneity and relies only upon advective transport. The purely advective model is able to explain a power-law trend of magnitude −2 to −3 in the breakthrough curve tail; a range that has been found in field tracer experiments. The principle cause of this trend is the comparatively slow transport in zones of small transmissivity (tight aperture). Slow advection occurs when either heterogeneity (variance of $\ln T$) is strong or when the assumed heterogeneity distribution is non-Gaussian. Thus, we link breakthrough tailing to the statistical parameters for the transmissivity field.

1. Introduction

When a non-reactive tracer is injected into a naturally or artificially flowing groundwater system, the late-time arrival of tracer at a detection point is extended with respect to the early arrival. This observation is referred to as a tracer breakthrough curve (BTC) tailing and often exhibits a power law decline over time. A −1.5 exponent of the tail is often observed in fractured media, which is generally attributed to matrix diffusion and/or sorption kinetics (e.g. Reimus et al., 2003; Cvetkovic et al., 2007). Exponents smaller than −1.5, in the range −3 to −2, have been also often found in the past (see, e.g., Becker and Shapiro, 2000, 2003; Kosakowski, 2004; more reference can be found in the literature review of Willmann et al., 2008), leading to the so-called "anomalous transport". As noted in the review by Zhou et al. (2007), considering only pore-scale matrix diffusion will lead to an underestimation of breakthrough tailing at the field scale, and explanations other than matrix diffusion have been offered in the past. The power-law BTC has been explained with many theoretical models including continuous time random walk (Berkowitz, 2006), mobile-immobile exchange (Haggerty and Gorelick, 1995; Haggerty et al., 2000), fractional-order advection/dispersion (Benson et al., 2000), among other approaches (Zhang et al., 2007, 2009). Most of those explanations invoke some local or non-local hydrodynamic dispersion and/or diffusion to help explain the behavior. By contrast, field examples have shown that power-law tailing can be observed in the absence of diffusion effects in highly heterogeneous velocity systems such as fractured bedrock (Becker and Shapiro, 2000, 2003). Recent direct numerical simulations (Wang and Cardenas, 2014) have shown a similar behavior at the core scale, observing an increasing deviation from the Gaussian behavior of the BTC for increasing fracture heterogeneity.

In this article, we examine the possibility that local dispersion, diffusion and the presence of eddies can be removed from the transport model entirely, and still account for strong tailing of the BTC similar to power-law. That is, power-law tailing with exponents in the range −3 to −2 in tracer BTC's can be explained by advection only. Similar problems were addressed by Wang and Cardenas (2014) through detailed numerical simulations, while our aim here is investigate them through a theoretical, physically based, analytical approach, which is more amenable to generalizations. The key to our examination is the high degree of heterogeneity often encountered in the subsurface, i.e. we consider a wide range in advective velocities that result from a highly heterogeneous conductivity field. We consider here fractured bedrock, in which advective velocities are expected to vary widely. The theoretical and mathematical construct developed here, however, is applicable to any highly heterogeneous transport system.

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The reason most often cited for the large variation in advective velocities in bedrock fractures is that flow is constrained to the two-dimensional domain of the fracture aperture. This invokes the "local cubic-law" (LCL) of fluid flow which dictates average flow rate at any location within a fracture is proportional to the cube of the aperture (Oron and Berkowitz, 1998). In this case, a local effective transmissivity (T) that relates local flow rate to local hydraulic gradient, is also expected to vary as the cube of the aperture. The LCL is not strictly valid under all circumstances. At large Reynolds or Peclet numbers (Detwiler et al., 2000; Konzuk and Kueper, 2004), in the presence of asperities or inclusions in the aperture that may create eddies (Konzuk and Kueper, 2004; Liu and Fan, 2012; Oron and Berkowitz, 1998; Qian et al., 2012) or under conditions of hydromechanical dilation and/or normal stress (Cornet et al., 2003; Gentier et al., 2013; Liu et al., 2013; Witherspoon et al., 1980), the rate of flow may not scale with the cube of the local aperture. However, for the development herein we do not need to invoke the LCL directly; we assume only small variations in aperture will cause large variations local advective velocity.

Fracture apertures are expected to vary from zero (wall contact) to about a mm in most competent bedrock systems. This variation can occur over the scale of cm, however, leading to highly variable velocity fields even within a single fracture. Reimus, for example found estimated 8 orders of magnitude in local permeability variation in tuff samples on the order of 10×10 cm in size (Reimus, 1995). The greater the normal stress on the fracture, the greater the contact area between the fracture walls (Gentier et al., 2013; Liu et al., 2013; Watanabe et al., 2008). Increased contact area (Watanabe et al., 2008; Witherspoon et al., 1980) and/or an increase in the coefficient of variation in aperture (Lee et al., 2003) will likely lead to an increase in hydromechanical dispersion and/or channeling.

There is unfortunately little field evidence of velocity distribution in bedrock fractures (Cvetkovic and Gotovac, 2013). Shapiro and Hsieh (1998) performed injection tests over short intervals in a crystalline granite/gneiss in central New Hampshire, USA, and found up to 5 orders of magnitude variation in transmissivity. Rutqvist et al. (1998) measured hydraulic apertures in granite between 8 and 164 microns, implying 4 orders of magnitude of T variability. Zhou et al. (2007) reviewed field tracer tests in fractured bedrock and found ranges in equivalent aperture ranging between 0.06 mm and 2.9 mm among different tests (implying 5 orders-of-magnitude variation in transmissivity). There is direct evidence that flow channeling occurs in bedrock fractures (Bourke, 1987; Moreno et al., 1985; Neretnieks, 1983, 1987; Neretnieks et al., 1982). In some cases, flow channeling has been imaged using ground penetrating radar (Becker and Tsoflias, 2010; Day-Lewis et al., 2006, 2003; Talley et al., 2005). The similarity between flow channeling observed in cores (e.g. Reimus, 1995; Watanabe et al., 2008) and in the field (e.g. Day-Lewis et al., 2002; Talley et al., 2005) suggests that velocity gradients can be large and occur at all scales in bedrock fractures.

Fracture apertures are often assumed to be lognormally distributed (Tsang and Neretnieks, 1998) but near gaussian distributions have also been measured (Bauget and Fourar, 2008; Lee et al., 2003). Konzuk and Kueper (2004) measured apertures in a fractured dolomite sample and initially found a lognormal distribution. After the distribution was de-trended, however, the apertures appeared to have a gaussian distribution. This suggests that other reported distributions may be affected by non-stationarity as well. For theoretical studies normal, log-normal, and tailed distributions (e.g. Painter, 2001; Painter et al., 2002) have been employed. Some authors have been able to reproduce laboratory breakthrough experiments by measure apertures and using small-perturbation theory to predict transport (Keller et al., 1999; Lee et al., 2003). However, these samples had relatively small coefficients of variation in aperture (between 0.4 and 0.82) and were conducted over short distances (16–30 cm) which may have contributed to the approximate agreement with small perturbation theory. Other researchers have suggested that Fickian processes are not sufficient to explain transport in fractures (Bauget and Fourar, 2008; Wang and Cardenas, 2014).

In the following we present a model of pure advection in a single, heterogeneous planar fracture. Flow heterogeneity derives from variable fracture aperture, which implies variable transmissivity (T). We employ a simplified but physically based analytical approach suited for highly heterogeneous fields. The model is therefore aimed at analyzing the BTC tailing and its relation to relevant measurable features of the transmissivity field.

2. Modelling approach

Similar to what done for aquifers (see, e.g., Dagan, 1989; Rubin, 2003), we model the medium as a stationary, two-dimensional (2D) random log-transmissivity ($Y = \ln T$) field, of distribution $f(Y)$, with mean $\langle Y \rangle = \ln T_G$ (the geometric mean), variance σ_Y^2 and isotropic two-point covariance; the transmissivity integral scale I is finite and is assumed to be much smaller than the characteristic length scale of the flow domain and of the solute plume. As stated in the Introduction, the variance σ_Y^2 can be very large in fractured systems, and thus a model suited to high heterogeneity is needed. The analytical model employed here is a 2D variant of the Self Consistent model (SCA) model which in the recent years was extensively analyzed and tested against accurate, large-scale numerical simulations (Jankovic et al., 2006; Fiori et al., 2006, 2007) as well as experimental data (Fiori et al., 2012, 2013). In this method, the first-passage travel time (t) distribution $f(t,x)$ is determined at a given control plane at a distance, x, from the source. For a non-reactive tracer $f(t,x)$ corresponds to the BTC at the same control plane. The calculation of the travel time distribution through the SCA method was originally proposed for 3D transport, and it is extended here for the first time to 2D flows. The main developments are briefly summarized in the following, and for further details of the general procedure developed for 3D flows the reader may refer to Fiori et al. (2006) or Cvetkovic et al. (2014).

The approach is to describe the transport medium as dense set of circular inclusions of radius R and random $Y = \ln T$ submerged in a homogeneous matrix of effective transmissivity T_{eff}. The velocity field in the heterogeneous medium is represented as the sum of the perturbation velocities associated with each isolated inclusion. The neglected nonlinear interactions among the inclusions are mimicked through the background effective transmissivity $T_{eff} = T_G$, the geometric mean on transmissivity (Dagan, 1989). Along these lines, the travel time t of a particle moving from $x = 0$ to x is the sum of travel time disturbances τ_{R_j} associated with the generic inclusion j, which can be written as

$$t = \langle t \rangle + \sum_{j=1}^{j=N} \tau_{Rj} \qquad (1)$$

where $\langle t \rangle = x/U$ is the mean and the residuals τ_{Rj} are independent random variables pertaining to the $N > 1$ blocks encountered by the particle. The random residual τ_R is derived by integration of the velocity field along the trajectory, as follows.

We consider uniform flow of velocity U at infinity, aligned with the longitudinal coordinate x_1, past a circular, isolated inclusion of radius R and transmissivity T, submerged in a medium of conductivity T_{eff}. The velocity field V is given exactly by Eq. (1) of Fiori et al. (2003), while the travel time residual τ_R is calculated by integration of dx/V_x, as described by Eq. (2) of Fiori et al. (2006).