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Review Paper

Remediation of pollution in a river by unsteady aeration with arbitrary initial and boundary conditions

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SUMMARY

The most important parameter controlling the water quality is the dissolved oxygen $Y(x,t)$ because it is very essential for aquatic life. An analytic solution is presented for unsteady equation representing the concentration of the dissolved oxygen $Y(x,t)$ along a river at any time t. The solution is obtained by using Laplace transformation technique. Adjoin solution techniques are used as boundary conditions to solve the equations. The variations of $Y(x,t)$ with time t from $t=0$ up to $t\to\infty$ (the steady state case) and with the parameters of the flow are taken into account in our study. It is shown that $Y(x,t)$ increases as t increases, keeping the other parameters constant, but $Y(x,t)$ decreases as the added pollutant rate along the river q increases. The adjoin solution techniques used in this work are effective and accurate for solving the equations representing the concentration of the dissolved oxygen $Y(x,t)$ when arbitrary initial and boundary conditions are given. The details are demonstrated in graphs.

Contents

1. Introduction

Stream temperature and dissolved oxygen *DO* are two critical factors affecting survival, movement and growth of fish (Beschta et al., 1987; Karim et al., 2003). During hot summer weather, high stream temperature and low dissolved oxygen problems often occur simultaneously and the resulting stress affects fish habitat use and survival (Matthews, 1998; Gouyuan, 2006). Unionid mussels elliptio crassidens were killed in Chickasawhatchee Creek, Baker Country, GA in July 2000, mainly due to the low flow velocity v(<0.01 m/s) and dissolved oxygen (<5 mg/L) (Gouyuan, 2006). Major fish kills also occurred due to the loss of the aquatic habitat. Actually, the evolution of the concentration of pollution $C(x,t)$ depends on the time t and the distance x as independent variables. At the same time, $C(x,t)$ depends on the temperature of water, acidity of water, pH, Dissolved oxygen, the flux of water, the quality of urban and industrial reject as parameters. The most important parameter is the dissolved oxygen $Y(x,t)$ because it is very essential for aquatic life. Without dissolved oxygen, a river would be an aquatic desert devoid of fish, plants and insects. For this reason, the concentration of the dissolved oxygen $Y(x,t)$ is studied in detail in this work. In spite of the fact that River Nile is artery of life in Egypt, unfortunately it is exposed to many kinds of chemical and biological pollutants in addition to the remains of the agricultural wastes. Here comes the importance of this study to know how we can predict the size of the pollutant concentration $C(x,t)$

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at the reasonable time as well as the procedure to control pollutants by increasing the concentration of the dissolved oxygen $Y(x,t)$. Impact of El-Rahawy Drain on the water quality of Rossetta Branch of the River Nile in Egypt is studied by Dimian et al. (2014). They determined the best zone from which water is taken for drinking or irrigation. Unsteady analytical solutions for one-dimensional advection-dispersion equation of the pollutant concentration are obtained by Wadi et al. (2014). They determined the parameters controlling the pollutant concentration along the river and generalized the earlier solutions obtained by Pimpunchat et al. (2007) for the unsteady state. The effect of added pollutant along a river on the pollutant concentration described by one-dimensional advection dispersion equation is studied by Dimian et al. (2013). They found that the five physical parameters controlling the pollutant concentration are reduced to only two dimensionless parameters, the dimensionless added pollutant concentration and the dimensionless dispersion.

The objective of this study is to develop analytical solution of one-dimensional unsteady state equation representing the concentration of the dissolved oxygen by using the method of Laplace transformations. The one-dimensional flow is justified for the reason that, in most natural streams, the longitudinal mass transport is more significant than lateral and vertical mass transport (Gouyuan, 2006). On the other hand, equations representing two dimensional flow can be transformed into one equation representing one- dimensional flow (Yadav et al., 2011). Our results generalize the earlier solutions obtained by Pimpunchat et al. (2007) for the concentration of the dissolved oxygen, which forms a subset of our solutions, for the limited case when $t \to \infty$.

2. Motivation

The motivation for the development of this type of solution is the need to predict the size of the dissolved oxygen concentration at reasonable time along a river. There are many problems related to the pollution of water. As an example, in the River Nile in Egypt, in summer 2012 many tons of fish were killed. At the same time, thousands of people got different diseases due to the water pollution (Dimian et al., 2014). The particular river whose water quality was the motivation for this study is the Tha Chin River in Thailand. The previous work published by Wadi et al. (2014), in the journal of earth system science presented analytical solutions for the unsteady advection dispersion equations describing the pollutant concentration $C(x,t)$ in one-dimension. On the other hand in the present work, an analytical solution is presented for unsteady equations representing the concentration of the dissolved oxygen $Y(x,t)$.

3. Formulation of the problem

Consider the unsteady flow in a river as being one-dimensional characterized by a single distance x(m) measured from the origin $x = 0$. The flow is described by the coupled equations for the pollutant concentration $C(x,t)$ (kg m^{-3}), and the dissolved oxygen concentration $Y(x,t)$ (kg m^{-3}), where t(day) denotes time. The dissolved oxygen concentration $Y(x,t)$ is important both for the survival of aerobic communities living in aquatic systems and also for the potential remediation of some of the unwanted pollutants by oxidation. Both $C(x,t)$ and $Y(x,t)$ are treated as homogeneous across the river cross section (Wadi et al., 2014; Pimpunchat et al., 2009). The two coupled advection–dispersion equations governing the flow can be written as (Pimpunchat et al., 2007).

$$\frac{\partial(AC)}{\partial t} = D_p\frac{\partial^2(AC)}{\partial x^2} - \frac{\partial(vAC)}{\partial x} - k_1\frac{Y}{Y+k}(AC) + qH(x), \quad x < L \leqslant \infty, t > 0, \tag{1}$$

$$\frac{\partial(AY)}{\partial t} = D_x\frac{\partial^2(AY)}{\partial x^2} - \frac{\partial(vAY)}{\partial x} - k_2\frac{Y}{Y+k}(AC) + \alpha(s_1 - Y), \quad x < L \leqslant \infty, t > 0, \tag{2}$$

where A is cross-section area (m^2); D_p is the dispersion coefficient of pollutant in the x-direction (m^2 day^{-1}); v is the water velocity in the x-direction (m day^{-1}); k_1 is the degradation rate coefficient for pollutant (day^{-1}); k is the half-saturated oxygen demand concentration for pollutant decay (kg m^{-3}); q is the added pollutant rate along the river (kg m^{-1} day^{-1}); D_x is the dispersion coefficient of dissolved oxygen in the x-direction (m^2 day^{-1}), taken as the same as D_p; k_2 is the de-aeration rate coefficient for the dissolved oxygen (day^{-1}); α is the mass transfer of oxygen from air to water (m^2 day^{-1}); s_1 is saturated oxygen concentration (kg m^{-3}); and $H(x)$ is the Heaviside function defined by:

$$H(x) = \begin{cases} 1, & 0 < x \\ 0 & \text{otherwise} \end{cases}.$$

Analytical solutions for Eq. (1), describing the pollutant concentration along the river are given by Wadi et al. (2014) for the special case when $k = 0$. The river has been divided into two regions: upstream $x \leqslant 0$ near the source, where it is assumed that the rate of pollutant along the river q(kg/(m d)) vanishes, the value of $Y(x,t)$ in this region will be denoted by $Y_1(x,t)$, and downstream $0 \leqslant x \leqslant L$(m) (the polluted length of the river), where $q = const$. The value of $Y(x,t)$ in this region will be denoted by $Y_2(x,t)$.

In this paper by taking $k = 0$, we can apply Laplace transformation and obtain solution for Eq. (2). The initial and boundary conditions associated with Eq. (2) are (Wadi et al., 2014; Pimpunchat et al., 2009):

$$Y_1(x,0) = Y_2(x,0) = Y_0, \text{say}, \tag{3}$$

$$Y_1(0,t) = Y_2(0,t), \quad t > 0, \tag{4}$$

$$\frac{dY_1(0,t)}{dx} = \frac{dY_2(0,t)}{dx}, \quad t > 0. \tag{5}$$

where Y_0 is the initial concentration of dissolved oxygen.

4. The analytical solution

The expressions for the pollutant concentration $C(x,t)$ in the two regions $x \leqslant 0$ and $x \geqslant 0$ are denoted by $C_1(x,t)$ and $C_2(x,t)$ respectively. For the steady state as $t \to \infty$, $C_1(x,t)$ and $C_2(x,t)$ are given by Wadi et al. (2014) and Pimpunchat et al. (2009)

$$C_1(x) = \frac{q}{k_1 A}\left(\frac{\beta-\delta}{2\beta}\right)\exp(\beta+\delta)x, \quad x \leqslant 0 \tag{6}$$

$$C_2(x) = \frac{q}{k_1 A}\left[1 - \left(\frac{\beta+\delta}{2\beta}\right)\right]\exp[-(\beta-\delta)]x, \quad x \geqslant 0 \tag{7}$$

where,

$$\delta = \frac{v}{2D_p} \quad \text{and} \quad \beta^2 = \frac{v^2}{4D_p^2} + \frac{k_1}{D_p}. \tag{8}$$

Applying Laplace transformation to Eq. (2) in the two regions $x \leqslant 0$ and $x \geqslant 0$ and using Eqs. (3), (6) and (7) gives:

$$\bar{Y}_1(x,p) = \frac{AY_0}{(Ap+\alpha)} + \frac{\alpha s_1}{p(Ap+\alpha)} + \frac{k_2 q}{2pk_1B^*}\left(\frac{\beta-\delta}{\beta}\right)x\exp[(\beta+\delta)x] + \alpha_1\exp\left[\left(\gamma + \sqrt{\frac{D_x\eta^2+p}{D_x}}\right)x\right], \quad x \leqslant 0 \tag{9}$$