



# Closed form flow model of a damped slug test in a fractured bedrock borehole



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## SUMMARY

An existing closed form model is modified to describe the damped response of groundwater in a fractured bedrock borehole with variable apertures and dips to a slug test. The existing theory, which requires single sized horizontal fractures, is accurately calibrated by slug test data from three uncased bedrock boreholes in the Dedham Granite and an observation well screened just below the contact surface with a till drumlin. Apertures and dips vary however, so the ability of the modified theory to accommodate different sizes and inclinations improves upon the physical validity of its predecessor when fracture information accompanies slug test data. Geophysical logs identify a large number and dip of fractures in the uncased boreholes in the Dedham Granite in this regard. A lognormally distributed, horizontal aperture calibration of the slug tests in the uncased boreholes retains the accuracy of the single size model, and yields aperture statistics more consistent with literature values. The slug test in the screened observation well is accurately calibrated with the modified horizontal theory for discrete (two) sizes, based upon the average fracture spacing found in the uncased boreholes. All four results yield comparable compressibility estimates, which depend on fracture spacing but not size or dip. The calibrated aperture size and calculated fracture porosity and permeability decrease with length of the borehole into the Dedham Granite. The measured dip and aperture for flowing and nonflowing fractures in one of the boreholes accurately calibrates the modified theory. The inclusion of dip reduces the calibrated permeability because of the increased ellipsoidal area at the interface of the borehole and the inclined fractures.

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## 1. Introduction

The hydraulics of bedrock fractures have been studied because of their importance to water supply wells (Siddiqui and Parizek, 1971), leakage around dams (Snow, 1970) and into tunnels (Mabee et al., 2002), radioactive repository flow (Follin et al., 2014) and transport (Cacas et al., 1990), and separate phase petroleum hydrocarbon withdrawal by fracking (Myers, 2012) and carbon dioxide injection after it is burned (Fang and Khaksar, 2013). Data sources range from tests on intact bedrock cores in the laboratory (Witherspoon et al., 1983) to geophysical logs (Boutt et al., 2010), strain measurements (Burbey et al., 2012), pump tests (Schweisinger et al., 2011), slug tests (Svenson et al., 2008), earth tides (Bredehoeft, 1967), and barometric pressure fluctuations (Rojstaczer and Agnew, 1989) in boreholes. Mapping of fracture networks in quarries (Lemieux et al., 2009) and natural outcrops (Manda et al., 2013) characterize bedrock at a larger scale.

Structural geology (DesRoches et al., 2014), network (Voeckler and Allen, 2012), finite difference (Bredehoeft and King, 2010),

and finite element models (Blessent et al., 2011) interpret regional hydraulics of fractured bedrock, while probability distributions describe the crossflow variability of apertures (Neuzil and Tracy, 1981), fracture spacing (Loiselle and Evans, 1995), porosity (Snow, 1968), and transmissivities (West et al., 2006) in individual boreholes. In the absence of geophysical logs, the probability density function of apertures responsible for these borehole integrated attributes is computed as a derived distribution calibrated by their observed behavior. Barker and Black (1983) and Shapiro and Hsieh (1998) use numerical Laplace transform inversion to model slug tests in boreholes with multiple horizontal fractures. Deterministic geomechanics models fracture formation (Boutt et al., 2009) and the coupled deformation and pressure response of a single size aperture to pump (Rutqvist et al., 1998; Schweisinger et al., 2009) and slug (Svenson et al., 2007) tests.

The present study combines elements of this substantial and continuing literature at the borehole scale by deriving a new closed form, deterministic model of slug test hydraulics in an idealized fractured bedrock borehole with varying apertures and dips, then using it with geophysical logs to interpret field data. Solutions are presented for discrete dips and apertures, and continuously distributed sizes. The theory is accurately calibrated with data,

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**Nomenclature**

$a$	largest aperture (L)	$r_B$	borehole radius (L)
$b$	second largest aperture (L)	$r_C$	casing radius (L)
$C_a$	integration constant for $a$ sized fracture (L T)	$S$	storativity of fractured bedrock borehole
$C_b$	integration constant for $b$ sized fracture (L T)	$T$	transmissivity of fractured bedrock borehole ( $L^2 T^{-1}$ )
$D_a$	hydraulic diffusivity of $a$ sized fracture ( $L^2 T^{-1}$ )	$t$	time (T)
$D_b$	hydraulic diffusivity of $b$ sized fracture ( $L^2 T^{-1}$ )	$u$	changed variable of integration
$E_a$	ratio of ellipsoidal to circular circumference for $a$ sized fracture	$\bar{v}_a$	average linear velocity in $a$ sized fracture ( $L T^{-1}$ )
$E_b$	ratio of ellipsoidal to circular circumference for $b$ sized fracture	$\bar{v}_b$	average linear velocity in $b$ sized fracture ( $L T^{-1}$ )
$F_T$	fracture porosity	$Y_N$	Bessel function of the second kind, of order $N$
$f(t):F(p)$	Laplace transform pair	$\alpha$	compressibility of fractured bedrock borehole ( $M^{-1} L T^2$ )
$g$	gravitational acceleration ( $L T^{-2}$ )	$\beta$	dimensionless time in Cooper et al. (1967) solution
$H$	disturbed hydraulic head at the borehole radius (L)	$\Gamma$	imaginary part of general slug test integrand
$H_O$	amplitude of slug test (L)	$\gamma_a$	imaginary part of Bessel function ratio for $a$ sized fracture
$H^*$	transformed hydraulic head at the borehole radius (L T)	$\gamma_b$	imaginary part of Bessel function ratio for $b$ sized fracture
$h$	disturbed hydraulic head (L)	$\Delta$	argument in integrand of Cooper et al. (1967) solution
$h^*$	transformed hydraulic head (L T)	$\theta_a$	dip of $a$ sized fractures
$J_N$	Bessel function of the first kind, of order $N$	$\theta_b$	dip of $b$ sized fractures
$K_N$	modified Bessel function of the second kind, of order $N$	$\kappa$	rate of rise of ambient piezometric surface ( $L T^{-1}$ )
$k$	permeability of fractured bedrock borehole ( $L^2$ )	$\nu$	kinematic viscosity of groundwater ( $L^2 T^{-1}$ )
$L$	borehole length (L)	$\rho$	density of groundwater ( $M L^{-3}$ )
$N_a$	number of $a$ sized fractures	$\sigma_E$	effective stress ( $M L^{-1} T^{-2}$ )
$N_b$	number of $b$ sized fractures	$\sigma_{\ln\phi}$	lognormal variance of aperture, with aperture in microns
$N_c$	number of $c$ sized fractures	$\phi$	aperture as a random variable (L)
$N_T$	total number of fractures	$\phi_I$	aperture with a cumulative density function of $I$ (L)
$n_a$	number fraction of $a$ sized fractures	$\phi_M$	mean aperture (L)
$n_b$	number fraction of $b$ sized fractures	$\phi_{50}$	median aperture (L)
$n_c$	number fraction of $c$ sized fractures	$\Omega$	real part of general slug test integrand
$p$	Laplace transform variable ( $T^{-1}$ )	$\omega_a$	real part of Bessel function ratio for $a$ sized fracture
$Q$	volumetric discharge in the radial direction ( $L^3 T^{-1}$ )	$\omega_b$	real part of Bessel function ratio for $b$ sized fracture
$R$	real variable of contour integration ( $T^{-1}$ )		
$r$	radial distance from center of borehole along fracture (L)		

logs, and stereonets from boreholes of increasing length in the Dedham Granite in eastern Massachusetts.

## 2. Theory

### 2.1. Governing equation

The conservation of incompressible fluid mass through a differential control volume in a uniform fracture balances storage and groundwater flux in the radial  $r$  direction in the fracture away from a borehole (Fig. 1).

$$\frac{\partial a}{\partial t} + \frac{a}{r} \frac{\partial(r\bar{v}_a)}{\partial r} = 0 \quad (1a)$$

$$\bar{v}_a = -\frac{a^2 g}{12\nu} \frac{\partial h}{\partial r} \quad (1b)$$

with time  $t$ . Witherspoon et al. (1980) demonstrate the validity of a laminar (parabolic) velocity profile across the aperture  $a$ , which implies the quadratic dependence of the average linear velocity  $\bar{v}_a$  in the fracture on its aperture. The disturbed hydraulic head is  $h$ , gravitational acceleration is  $g$ , and kinematic viscosity of the groundwater is  $\nu$ . Groundwater does not flow through the solid matrix around the fracture, nor leak into the fracture in our model. In this regard, secondary fractures, which would leak groundwater into the control volume and complicate the inversion of the transformed transport equation (Barker and Black, 1983), are ignored. The aperture does not vary with  $r$ , as considered in the coupled finite difference model of Svenson et al. (2007), nor with

circumferential distance, as analyzed by Neuzil and Tracy (1981) for a single fracture. Eq. (1) ensures that the radial variation of hydraulic head and the groundwater discharge through the fracture depend on the aperture. This complicates the interpretation of integrated properties like transmissivity and permeability of the fractured bedrock borehole, as noted by Shapiro and Hsieh (1998).

Each of the  $N_T$  fractures in the borehole length  $L$  is assumed to deform equally and elastically to a change of effective stress  $d\sigma_E$  regardless of its aperture or dip  $\theta_a$ . The linearizing assumption implies that the change of effective stress is small, distinguishing slug tests from high pressure injection tests (Rutqvist et al., 1998). Dip, stress, and aperture are linked for large changes of stress, and more sophisticated models are needed to describe the coupled response of the fractured bedrock and fluid in this case. The assumption of a constant total stress and the chain rule cast storage in the aperture as a function of hydraulic head, the fracture compressibility  $\alpha$ , the fractured bedrock borehole storativity  $S$ , and  $N_T$

$$\alpha = -\frac{N_T da}{L d\sigma_E} \quad (da = db..) \quad (2a)$$

$$\frac{\partial a}{\partial t} = \frac{S}{N_T} \frac{\partial h}{\partial t} \quad (d\sigma_E = -\rho g dh) \quad (2b)$$

$$S = \rho g \alpha L \quad (2c)$$

$$N_T = N_a + N_b + \dots \quad (2d)$$

The groundwater density is  $\rho$  and the borehole spacing for all fractures  $L/N_T$  defines the incremental strain for all fractures in Eq. (2a). The constraint anticipates at least one other (smaller) aperture  $b$  in the borehole: every fracture releases the same

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