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Performance of the analytical solutions for Taylor dispersion process in open channel flow

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SUMMARY

The present paper provides a systematical analysis for concentration distribution of Taylor dispersion in laminar open channel flow, seeking fundamental understandings for the physical process of solute transport that generally applies to natural rivers. As a continuation and a direct numerical verification of the previous theoretical work (Wu, Z., Chen, G.Q., 2014. Journal of Hydrology, 519: 1974–1984.), in this paper we attempt to understand that to what extent the obtained analytical solutions are valid for the multi-dimensional concentration distribution, which is vital for the key conclusion of the so-called slow-decaying transient effect. It is shown that as a first estimation, even asymptotically, the longitudinal skewness of the concentration distribution should be incorporated to predict the vertical concentration correctly. Thus the traditional truncation of the concentration expansion is considered to be insufficient for the first estimation. The analytical solution by the two-scale perturbation analysis with modifications up to the second order is shown to be a most economical solution to give a reasonably good prediction.

1. Introduction

Taylor dispersion process (Taylor, 1953) generally refers to an asymptotic stage of the solute transport in a transversely confined flow region: During the process, the cross-sectional mean concentration longitudinally forms a Gaussian distribution, which is governed by a diffusion equation with the "diffusivity" (also known as Taylor dispersivity) exceeding corresponding molecular diffusivity (or turbulent diffusivity) for several orders of magnitude. Since the discussed configuration is generally encountered (for environmental processes it is found in rivers, estuaries, oceans and atmosphere, for example) (Cassol et al., 2009; Fischer, 1976; Fischer et al., 1979; Ngo-Cong et al., 2015; Steinbuck et al., 2011; Wang and Chen, 2015; Wu, 2014; Wu and Chen, 2014a; Wu et al., 2012; Zeng et al., 2014; Wu et al., 2015), and the phenomenon is so remarkable due to a so-called macroscopic mechanism of "enhanced diffusion", the concept of Taylor dispersion has attracted intensive studies and found extensive applications in past decades (Ajdari et al., 2006; Aris, 1956, 1959; Chen et al., 2012; Ng and Chen, 2013; Ng and Zhou, 2012; Smith, 1982, 1983; Stone and Brenner, 1999; Stone et al., 2004; Wu and Chen, 2014b; Yasuda, 1984).

Taylor dispersion has traditionally been recognized as a one-dimensional process: it is the cross-sectional mean concentration that has been mainly studied (Taylor, 1953; Wu, 2014; Wu and Chen, 2014b). In Taylor's initiative, the diffusion equation for the mean concentration was for the first time proposed, with the Taylor dispersivity analytically determined and corresponding results experimentally verified by the scalar transport process in a long and thin tube (Taylor, 1953). The followed experimental explorations had also shown the validity of the theory for the transport process in gas (Bournia et al., 1961; Evans and Kenney, 1965). To analytically tackle the problem at a different perspective, Aris resorted to the method of concentration moment (Aris, 1956) to describe the statistical behavior of the concentration evolution, which has now become a classic technique in dealing with Taylor dispersion process. Although the convection–diffusion equation governing the entire transport process cannot be exactly solved, the solvable resulted moment equations at different orders provide accurate information including but not limited to the displacement of the centroid, the variance, and the skewness of the solute cloud, depending on the specific order of the equation. However, as indicated by later researchers, the moments do not directly result in a distribution of the concentration (Phillips and Kaye, 1996, 1997). And different analytical methods were then proposed for the mean concentration evolution such as Gill's method of mean concentration expansion (Chen, 2013; Gill, 1967; Gill and Sankarasubramanian, 1970), delay-diffusion equation (Smith, 1981), homogenization technique (Mei et al., 1996; Ng and Yip, 2001), the analysis based on the Lyapunov-Schmidt

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technique (Ratnakar and Balakotaiah, 2011), and some other asymptotic analysis (Latini and Bernoff, 2001; Phillips and Kaye, 1996; Stokes and Barton, 1990). There are also some numerical attempts for the entire process of scalar transport, the results of which provided information for the mean concentration evolution not only on the Taylor dispersion process, but also on the pre-stage process dominated by the mechanism of convection (Ekambara and Joshi, 2004; Houseworth, 1984; Stokes and Barton, 1990).

To understand the various natural processes associated with Taylor dispersion, e.g. contaminant transport in rivers, information on the concentration cloud not only for the cross-sectional mean but also for the transverse distribution details should be acquired (Wu and Chen, 2014a). The existing viewpoint on the transverse concentration distribution, though without a direct validation, is that the transverse pattern can be ignored, or considered as uniform, after the time scale when Taylor dispersion model becomes applicable (Latini and Bernoff, 2001; Stokes and Barton, 1990; Taylor, 1953). However, by a recent analytical study with the proposed two-scale perturbation analysis for Taylor dispersion process (Wu and Chen, 2014b), it is shown that the two processes cannot be regarded as the same one with respect to the characteristic time-scale, respectively for approaching longitudinal normality of the mean concentration and approaching uniformity of the transverse concentration. The latter process is shown to last much longer than the former, even by orders of magnitude, and is thus regarded as the slow-decaying transient effect (Wu and Chen, 2014a).

Taylor dispersion in laminar flows such as the laminar open channel flow is one of the most idealized processes and of great importance, since it provides fundamental understandings for the transport process that generally applies to more complicated flow regimes such as the turbulent flows (Wu, 2014; Wu and Chen, 2014a). The related investigations themselves can also be directly extended for more complex transport configurations. A classic example is the extension of Taylor dispersion in laminar tube flow to that in turbulent tube flow (Taylor, 1953, 1954), and then in natural rivers and estuaries (Fischer, 1969, 1973; Fischer and Hanamura, 1975; Fischer and Holley, 1971; Fischer et al., 1979; Pannone, 2014; Smith, 1987). For the present laminar open channel flow, analytical attempt by the two-scale perturbation analysis to address the transverse concentration distribution and evolution has been preliminarily carried out (Wu and Chen, 2014a). Further application of the analytical technique for solute transport in turbulent open channel flow is also shown to be possible and straightforward (Wu, 2014). However, here we still focus on the laminar flow case to provide essential information for the physical process of solute transport.

By now, all the previous explorations on the transverse concentration distribution as indicated above are heavily dependent on the analytical solutions, and no systematical study is complemented to address the performance of these solutions (Wu, 2014; Wu and Chen, 2014a,b). The only validation is performed by comparing the mean concentration between the analytical and existing numerical results for solute transport in the laminar tube flow (Wu and Chen, 2014b). But a reasonable concern is that a good approximation for the mean concentration does not necessarily lead to a proper prediction for the transverse distribution. Thus it remains unclear that to what extent the results by the analytical solution may deviate from the real concentration distribution, which is extremely important for validating the key conclusion of the slow-decaying transient effect. For the applied two-scale perturbation analysis, concerns also have been raised for the convergence of the analytical solutions on that, how many perturbation problems (or orders of modifications on the zeroth-order solution) are needed in obtaining solutions for a high-precision and a reasonably good prediction, respectively aiming at possibly studying the transport mechanism and providing an easy-to-use formula for engineering purpose. All in all, a systematic study is needed to evaluate the performance of the obtained analytical solutions.

In this paper, we first continue to deduce the third-order modifications for the zeroth-order solution by the two-scale perturbation analysis. Then the analytical solutions with modifications to the Gaussian distribution up to the first-, the second-, and the presently obtained third-order are to be analyzed, with reference to a traditional truncation of the conventional concentration expansion for the two-dimensional concentration distribution, by comparison with some present results of numerical simulations. Discussions and conclusions are to be provided at the last part of the paper.

2. Method and materials

When the solute is released into the open channel flow, there are generally two phases for the concentration transport process. The first one is dominated by the flow convection, and the solute is generally advected by the fluid flow with different speeds at different vertical positions. After a time scale of H^2/D, where H is the depth of the channel and D the molecular diffusivity (or turbulent diffusivity for turbulent flow), the mechanism for the longitudinal scatter of the cloud is dominated by Taylor dispersion, which is contributed by the combined action of flow shear and vertical diffusion. The flow shear stretches the solute cloud to cause its longitudinal separation and the vertical concentration difference, the latter of which is at the same time smeared out by the vertical diffusion. As a result, the centroid of the solute cloud moves with the cross-sectional mean velocity of the flow, and the evolution of the longitudinal distribution of the vertical mean concentration resembles the one-dimensional molecular diffusion, though with a much greater “diffusivity” (Taylor dispersivity). Under typical conditions, the Taylor dispersivity can exceed the molecular diffusivity (or turbulent diffusivity) for several orders of magnitude, and thus it is also referred to as a mechanism of “enhanced diffusion”.

In the previous study (Wu and Chen, 2014a), we have already obtained the analytical solution for the multi-dimensional concentration distribution by the two-scale perturbation analysis with modifications to the Taylor dispersion model up to the second order, $\Omega_{(2)}$. In addition to refer to the deduced solutions in this paper, $\Omega_0 - \Omega_{(2)}$, we continue to consider higher order perturbation problems to obtain the analytical solution with modifications up to the third order, $\Omega_{(3)}$. And besides, some existing form of the first estimation for the multi-dimensional concentration distribution, $\Omega_{(00)}$, is also introduced for analysis. To systematically evaluate the performance of the analytical solutions, in this paper we perform numerical simulations under typical parameters, and the analytical and numerical results are cross-checked and combined to illustrate the limit of the traditional first estimation for the concentration distribution. The structure of the present paper is shown in Fig. 1.

A sketch is given as Fig. 2 to illustrate the configuration of laminar open channel flow. The channel is of depth H. In a Cartesian coordinate system, x-axis defines the longitudinal direction, z-axis the vertical direction, and O at the channel bed wall is the origin. The flow velocity u is only a function of z.

The solute transport process can be exactly governed by an advection–diffusion equation as

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} = D\frac{\partial^2 C}{\partial x^2} + D\frac{\partial^2 C}{\partial z^2}, \tag{1}$$

where C is concentration, t time, u velocity distribution, and D the molecular diffusivity.