Journal of Hydrology 529 (2015) 398–409

Contents lists available at ScienceDirect

Journal of Hydrology

journal homepage: www.elsevier.com/locate/jhydrol

A general approximate method for the groundwater response problem caused by water level variation

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ARTICLE INFO

Article history:
Received 8 March 2015
Received in revised form 24 June 2015
Accepted 20 July 2015
Available online 4 August 2015
This manuscript was handled by Corrado Corradini, Editor-in-Chief, with the assistance of Rao S. Govindaraju, Associate Editor

Keywords:
Boussinesq equation
Unconfined aquifer
Moving boundary
Approximation method

SUMMARY

The Boussinesq equation (BEQ) can be used to describe groundwater flow through an unconfined aquifer. Based on 1D BEQ, we present a general approximate method to predict the water table response in a semi-infinite aquifer system with a vertical or sloping boundary. A decomposition method is adopted by separating the original problem into a linear diffusion equation (DE) and two correction functions. The linear DE satisfies all the initial and boundary conditions, reflecting the basic characteristics of groundwater movement. The correction functions quantitatively measure the errors due to the degeneration from the original BEQ to a linear DE. As the correction functions must be linearized to obtain analytical solution forms, the proposed method is an approximate approach. In the case studies, we apply this method to four different situations of water level variation (i.e., constant, sudden, linear and periodic change) resting on vertical or sloping boundaries. The results are compared against numerical results, field data and other analytical solutions, which demonstrate that the proposed method has a good accuracy and versatility over a wide range of applications.

1. Introduction

Groundwater flow in an unconfined aquifer can be modeled by the nonlinear Boussinesq equation (BEQ). The solutions of the BEQ predict the response of the groundwater table in an unconfined aquifer due to stream water variations and quantify the exchange flow between the stream and aquifer. The results are useful in situations such as slope stability, irrigation, drainage and catchment hydrology.

Of special interest are groundwater flow and bank storage effects caused by changes in water elevation in free water bodies adjacent to the aquifer (Govindaraju and Koelliker, 1994). The situations of water level variation depend on many factors, such as the geometry of stream cross-section, characteristics of the drainage basin, time–space variation of storms, snowmelt and rainfall runoff. Analytically solving such problem can provide insights into the physical processes of water recharging and dewatering (Lockington et al., 2000; Song et al., 2007; Basha, 2013; Workman et al., 1997; Ostfeld et al., 1999; Liang and Zhang, 2012).

The difficulties in obtaining analytical solutions for groundwater table response problem are from three aspects. The first is that variable water heights make the boundary condition dynamic. To obtain analytical solutions, the original problem was simplified to linearized models with vertical interfaces. For instance, for the problem in an infinite region, Govindaraju and Koelliker (1994) and Basha (2013) assumed a general mode of water level variation, and Parlange et al. (2000) adopted a special function for describing water variation so that his polynomial solution can be matched. Song et al. (2007) and Lockington et al. (2000) proposed their analytical solutions to the case in which water level varies by a power function. Moutsopoulos (2013) and Teloglou and Rajeev (2012) studied the cases where the boundary conditions are of the Robin and Cauchy type, and the modes of water level variation are also different. For the flow in a finite region, Kim and Ann (2001) discussed a case with water heads fixed at both ends. Workman et al. (1997) set up a model by assuming a head fixed at one end but varying with respect to time at the other. Serrano and Workman (1998) extended this model by allowing variable heads at both ends.

The second difficulty is from the so-called moving boundary effects (Li et al., 2000). When the water varies on a sloping boundary, the heads acting on the slopes may vary along both the time and space dimensions. The moving boundary effect increases the nonlinearity of the BEQ, which leads to that the analytical solutions are difficult to obtain. Nielsen (1990) was probably the first to examine this problem. He obtained an analytical solution in a study of aquifer response to ocean tides. However, his solution

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http://dx.doi.org/10.1016/j.jhydrol.2015.07.030
0022-1694/

failed to satisfy the boundary condition precisely because the assumption of a fixed location of the shoreline boundary condition is relaxed. Li et al. (2000) revisited this case and addressed the problem by using a coordinate transformation to convert the original BEQ to an advection–diffusion equation. By using perturbation method, both Nielsen's and Li's solutions are applicable in the case of small water level fluctuations. Sun et al. (2011) and Zheng et al. (2005) studied a moving boundary case with a constant speed of water level variation. Zheng et al. (2005) simplified the sloping boundary to a vertical case. By following Li's coordinate transformation concept, Sun et al. (2011) obtained their approximate solutions for predicting the groundwater table variation with a sloping boundary. However, the approximate solutions are in the form of upper and lower bounds of the exact solution. All those studies are based on the linearized BEQ.

Third, to the authors' knowledge, there is no general method capable of handling the nonlinear BEQ problems, especially ones associated with dynamic and moving boundary effects. The most common method is through linearization, whereby the head function in the coefficients of BEQ is replaced by a constant characteristic head (Govindaraju and Koelliker, 1994). The solutions of the linearized equation are then obtained by various approximate methods to achieve applicable results, such as perturbation methods (Parlange et al., 1984; Nielsen, 1990; Li et al., 2000; Teo et al., 2003; Song et al., 2007; Roberts et al., 2011), Laplace transformation methods (Marino, 1975; Ostfeld et al., 1999; Rai and Manglik, 1999; Akylas and Koussis, 2007; Bansal, 2012), weighted residual methods (Lockington, 1997; Lockington et al., 2000), variable separation methods (Workman et al., 1997; Kim and Ann, 2001), Boltzmann transformation and decomposition methods (Chor et al., 2013) and decomposition method (Serrano and Workman, 1998). Lacking generality, the above-mentioned methods would be more suitable for particular cases. More recently, some promising analytical techniques have emerged, such as Adomian's decomposition (Moutsopoulos, 2013; Serrano et al., 2007), the traveling wave solution (Basha, 2013), Homotopy Perturbation Method (Ganji et al., 2011) and the Homotopy Analysis Method (HAM) (Song and Tao, 2007). However, these methods involve advanced mathematical theories that are unfamiliar to most engineers or earth scientists and their applications are therefore somewhat limited.

This paper proposes a general approximate method to predict aquifer response subject to water level variations in a free water body. It is applicable to both the linearized and nonlinear BEQ. This proposed method decomposes the original nonlinear PDE into a linear diffusion equation (DE) and two nonlinear correction functions. Satisfying the initial and boundary conditions, the DE has analytical solution, reflecting the basic characteristics of groundwater movement. The correction functions use its solution as the basis to measure the errors caused by the reduction from the nonlinear BEQ to a linear DE. Because the correction functions are nonlinear PDEs, they must be linearized to obtain the approximate analytical solutions. Thus, the proposed method is an approximation approach.

The paper is organized as follows. Focusing on a semi-infinite aquifer with a sloping interface, we give mathematical descriptions of the groundwater response caused by variable water level in Section 2. A general analytical method is proposed thereafter. In Section 3, an adaptive finite volume method (FVM) is introduced to solve the moving boundary problem. In Section 4, the proposed analytical method is applied to four different situations of water level variation (i.e., constant, sudden, linear and periodic change). The solutions are compared to the numerical results from the FVM and other analytical solutions to verify their validity. Section 5 discusses the applicability conditions of the analytical method. Conclusions are drawn in the final section.

2. Mathematical model and approximate solution

The BEQ describes the transient groundwater movement in an unconfined aquifer. It reads as follows:

$$\frac{\partial h}{\partial t} = \frac{K}{S}\frac{\partial}{\partial x}\left(h\frac{\partial h}{\partial x}\right) \tag{1}$$

where h is the water depth above an impervious substratum [L]; K is the hydraulic conductivity [LT^{-1}]; S is the specific yield of the aquifer; x is the horizontal coordinate [L]; and t is the time step [T]. Eq. (1) does not include the effects of capillary action and replenishment from the rainfall infiltration. The aquifer is assumed to be homogeneous, so K and S are taken as constants.

The initial and boundary conditions are assumed as follows:

$$h(x,0) = h_i \tag{2}$$

$$h(X(t),t) = h_i - \int_0^t \upsilon(\tau)d\tau \tag{3}$$

$$h(\infty,t) = h_i \tag{4}$$

where h_i (>0) is the initial height of the groundwater table; $\upsilon(t)$ is the velocity function of water level variation, assumed to be positive when reservoir water drops (i.e., Fig. 1a) and negative when reservoir water rises (i.e., Fig. 1b); and $X(t)$ is the position of the moving boundary on the x axis, namely $X(t) = [h_i - s(t)]\ \cot(\theta)$, where θ is the slope angle and $s(t) = \int_0^t \upsilon(\tau)d\tau$. For the water drawdown condition, the effect of the seepage face is ignored.

Similar to Li et al. (2000), we introduce a new variable $z = x - X(t)$. Eqs. (1)–(4) can be restated as follows:

$$\begin{cases} \frac{\partial h}{\partial t} = \frac{K}{S}\frac{\partial}{\partial z}(h\frac{\partial h}{\partial z}) - \upsilon(t)\cdot\cot(\theta)\frac{\partial h}{\partial z} \\ h(0,t) = h_i - s(t), \quad h(\infty,t) = h_i, \quad h(z,0) = h_i \end{cases} \tag{5}$$

Eq. (5) is the mathematical description of groundwater movement due to the water level variation on a sloping boundary. It is nonlinear without an exact solution in general. If we let $\alpha = Kh_i/S$, $\gamma = K/S$ and substitute h_i - $d(z, t)$ - $\delta(z, t)$ for $h(z, t)$, Eq. (5) can be decomposed as follows:

$$\begin{cases} \frac{\partial d}{\partial t} = \alpha(\frac{\partial^2 d}{\partial z^2}) - \upsilon(t)\cdot\cot(\theta)\frac{\partial d}{\partial z} \\ d(0,t) = s(t), \quad d(\infty,t) = 0, \quad d(z,0) = 0 \end{cases} \tag{6}$$

and

$$\begin{cases} \frac{\partial \delta}{\partial t} = \alpha\left(\frac{\partial^2 \delta}{\partial z^2}\right) - \gamma\frac{\partial}{\partial z}\left((d+\delta)\frac{\partial(d+\delta)}{\partial z}\right) - \upsilon(t)\cdot\cot(\theta)\frac{\partial \delta}{\partial z} \\ \delta(0,t) = 0, \quad \delta(\infty,t) = 0, \quad \delta(z,0) = 0 \end{cases} \tag{7}$$

Eq. (6) is essentially the linearized form of Eq. (5) because the head function h in the term $h\frac{\partial h}{\partial z}$ is characterized by a constant head h_i. As the coefficient $\upsilon(t)$ is a function, Eq. (6) is difficult to solve analytically. Its solution may exist only under certain special conditions, such as θ is equal to $\pi/2$, or $\upsilon(t)$ is a constant. Although Eq. (6) is still mathematically nonlinear, we refer to it as the linearized BEQ to distinguish it from the original Eq. (5). Furthermore because Eq. (7) is the result of the degeneration from Eqs. (5) and (6), it is called the head correction hereafter because its role can be thought of as the compensation for the errors arising from the linearization process.

2.1. The solution for the linearized BEQ

Substituting $d_1(z, t) + d_2(z, t)$ for $d(z, t)$ in Eq. (6), we can further decompose it to a diffusion equation $d_1(z, t)$ and another correction function $d_2(z, t)$. That is,