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The use of electrical conductivity measurements in the prediction of hydraulic conductivity of unsaturated soils

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S U M M A R Y

Statistical models have been widely used in soil science, hydrogeology and geotechnical engineering to predict the hydraulic conductivity of unsaturated soils. However, no effective method is available yet for the determination of the associated model parameters such as the tortuosity factor q. Considering the analogy between water flow and electrical current flow in a porous medium, in this study, we proposed to improve the predictive capability of statistical models by determining the tortuosity factor q using electrical conductivity (*EC*) measurements. We first developed a theoretical hydraulic–electrical conductivity (*K*–*EC*) relationship for unsaturated soils based on the bundle of capillary tubes model. This *K*–*EC* relationship was then used to form a new unsaturated soil *EC* model, which was verified using published experimental data. The tortuosity factor q can then be determined by fitting the new *EC* model to soil *EC* measurements. Experimental data of six soils were used to test the effectiveness of this method and it was shown that the prediction was significantly improved when compared with the one using the commonly suggested value $q = 0.5$. The associated root-mean-square-deviation (*RMSD*) between measurements and predictions is only 0.28 when q is obtained by using our proposed method. In contrast, the *RMSD* is 0.97 when q is simply assumed as 0.5.

1. Introduction

Hydraulic conductivity is essential in modeling the water flow and transport processes in the subsurface (Tindall et al., 1999; Lu and Likos, 2004; Fredlund et al., 2012). The hydraulic conductivity of a soil varies over several orders of magnitude when the soil becomes unsaturated, and the magnitude of the unsaturated hydraulic conductivity K is actually a function of the suction head h or volumetric water content θ. To determine K, the most fundamental and straightforward way is to conduct measurements on disturbed or undisturbed soil samples in the laboratory using the steady-state method (Barden and Pavlakis, 1971; Klute, 1972; Huang et al., 1998) or the transient method (Hamilton et al., 1981; Meerdink et al., 1996; Fujimaki and Inoue, 2003; Li et al., 2009). The measurement of K can also be conducted in situ (Bouma et al., 1971; Ankeny et al., 1991; Benson and Gribb, 1997; Zhang et al., 2000) so that the soil heterogeneity and (or) anisotropy can be accounted for. Although these methods have been widely applied in practice, in most cases, the K measurement is difficult to set up, time-consuming and error prone.

Alternatively, K can be estimated using pedotransfer functions (Bouma, 1989), which physically or empirically relate K to some easily measured soil properties, e.g., soil texture (Alexander and Skaggs, 1987; Wösten and van Genuchten, 1988; Wösten et al., 2001) and soil water retention curves (*SWRCs*) (Childs and Collis-George, 1950; Burdine, 1953; Campbell, 1974). Among them are statistical models (Mualem and Dagan, 1978), which have been widely used in soil science and hydrogeology (Mualem and Klute, 1986; Durner, 1994; Schaap and Leij, 2000; Peters and Durner, 2008). The statistical models utilize the *SWRC* to infer the pore size distribution of the soil and thus to predict the unsaturated hydraulic conductivity. A general expression of statistical models can be expressed as follows,

$$K = K_{sat} \cdot K_r = K_{sat} S_e^q \left(\frac{\int_0^{\theta} \frac{1}{y(x)^{\beta}} dx}{\int_0^{\theta_s} \frac{1}{y(x)^{\beta}} dx} \right)^{\gamma} \quad (1)$$

where K_{sat} is the saturated hydraulic conductivity; K_r is the relative hydraulic conductivity; $y(x)$ is the *SWRC* of the soil; S_e is the relative saturation, in the form of $(\theta - \theta_r)/(\theta_{sat} - \theta_r)$; θ_{sat} and θ_r are the saturated and residual volumetric water contents of the soil,

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respectively. The parameter q, i.e., the tortuosity factor, is a lumped parameter that accounts for pore tortuosity and pore connectivity. In this paper, we consider Mualem (1976)'s model in which $\beta = 1$ and $\gamma = 2$.

Based on experimental results from 45 different types of soils, *Mualem* [1976] suggested $q = 0.5$ for sands, loams and other soils with stable structure. This value has been frequently used in practice and has been experimentally verified for a number of soils (e.g., see van Genuchten, 1980; Othmer et al., 1991; Durner, 1994; Vandam et al., 1994; Kosugi, 1996; Mohanty et al., 1997; Tuller and Or, 2001; Peters and Durner, 2008). However, it has also been found that allowing q to vary among different soils gives a better description of the measured unsaturated hydraulic conductivity (Mualem, 1976; Schuh and Cline, 1990; Yates et al., 1992; Weerts et al., 1999, 2001; Schaap and Leij, 2000). A number of researches have revealed that correlations exist between q and some other soils properties, e.g., particle diameter (Schuh and Cline, 1990), soil water retention parameters (Schaap and Leij, 2000) and saturated hydraulic conductivity (Vervoort and Cattle, 2003). In spite of these findings, it is still not yet possible to directly and reliably estimate q from other soils properties. To improve the predictive capability of statistical models, e.g., Mualem (1976)'s model, further efforts, therefore, are needed to develop a method that can easily and effectively determine the tortuosity factor q.

In porous media, water flow and electrical current flow are analogous processes, and both of them are influenced by the same parameters, e.g., porosity, tortuosity, etc. As electrical conductivity (*EC*) is an easily measured soil property, it is beneficial to use soil *EC* measurements as an aid to the prediction of hydraulic conductivity of unsaturated soils (Mualem and Friedman, 1991; Doussan and Ruy, 2009), e.g., in determining the tortuosity factor q in Mualem (1976)'s model. Indeed, Mualem and Friedman (1991) have related the tortuosity factor q to the soil *EC* in developing their *EC* model for unsaturated soils (referred to as the *MF* model hereinafter). Intuitively, it seems possible to determine q by fitting the *MF* model to the measured soil *EC* data. Although this idea appears to be feasible and plausible, it has not yet been explored and tested, perhaps because of two inherent limitations of the *MF* model. Firstly, it is assumed that reducing water saturation S had an identical effect on the decrease in soil electrical and hydraulic conductivities (Mualem and Friedman, 1991). However, it is still unclear whether this assumption is theoretically valid (to be discussed later). Secondly and more importantly, the *MF* model cannot properly describe the electrical conductivity of soils close to fully saturation, and the prediction from this model behaves unrealistically after exceeding a certain water content (Weerts et al., 1999) (also to be discussed later). To apply this model, special treatment therefore is needed in practice (Vanclooster et al., 1994; Weerts et al., 1999). Considering these limitations, it is necessary to develop a new *EC* model for unsaturated soils that can incorporate the tortuosity factor q.

The first objective of this study, therefore, is to develop such an *EC* model for unsaturated soils. We first develop a theoretically-based hydraulic–electrical conductivity (*K–EC*) relationship for unsaturated soils. This *K–EC* relationship and the *K* model proposed by Mualem (1976) are combined to form a new *EC* model for unsaturated soils. Afterwards, the new *EC* model is tested experimentally using the published data for four different porous media. The second objective of this study is to test the idea of using soil *EC* data as an aid in the prediction of the unsaturated hydraulic conductivity. We demonstrate the detailed procedures of using the new soil *EC* model to determine the tortuosity factor q in order to improve the predictive capability of the Mualem (1976)'s model. We also provide experimental verifications for this improved *K* prediction. In the last part of this paper, some practical issues regarding this proposed method are discussed.

2. *K–EC* relationship

Because of the potential to predict hydraulic conductivity from electrical conductivity measurement, many researches have been done to study the hydraulic–electrical conductivity (*K–EC*) relationship for saturated porous materials in the past several decades (e.g., Urish, 1981; Purvance and Andricevic, 2000; Slater and Lesmes, 2002). Recently, similar approaches have been extended to unsaturated porous materials (e.g., Doussan and Ruy, 2009), and some unified *K–EC* relationships have been proposed (see Revil et al., 2014). In this section, we theoretically form a hydraulic–electrical conductivity (*K–EC*) relationship for unsaturated soils using a bundle of capillary tubes model. It should be noted that the electrical conductivity here is the contribution from the pore water. Although in this study we use the term, *K–EC* relationship, it actually refers to the relationship between the hydraulic conductivity and electrical conductivity of unsaturated soils that are contributed from the pore water.

The capillary tubes model geometrically is not a good representation of the pore space in most porous media (Doussan and Ruy, 2009). For example, it cannot account for the irreducible water content in porous materials, and it also cannot model the bypass flow between neighboring capillary tubes. Nevertheless, it still can provide valuable insights into different transport phenomena occurring in porous media from the microscopic viewpoint (e.g., van Dijke et al., 2001; Jackson and Blunt, 2002). In addition, all macroscopic parameters, e.g., the volumetric water content, electrical and hydraulic conductivities, can be directly formalized from the model by simply analyzing the capillary tubes. Because of these merits, the bundle of capillary tubes model has been extensively used and has achieved great success in, for example, calculating the dynamic effects in the capillary pressure–saturation relationship (Celia et al., 2004; Dahle et al., 2005), characterizing the multiphase electrokinetic coupling in porous media (Jackson, 2008, 2010) and describing the water flow in frozen soils (Watanabe and Flury, 2008).

Fig. 1 shows a soil element with a length of L and a cross section area of A. The connected pores inside the soil element are simulated by a bundle of capillary tubes with different radii. We assume that pore water only flows through these capillary tubes and there is no bypass flow among neighboring capillary tubes. In addition, we consider the radius r of the capillary tubes (i.e., the pore size) as a random variable that can be described by a probability density function $f(r)$. The cross section area of each capillary tube A_c is

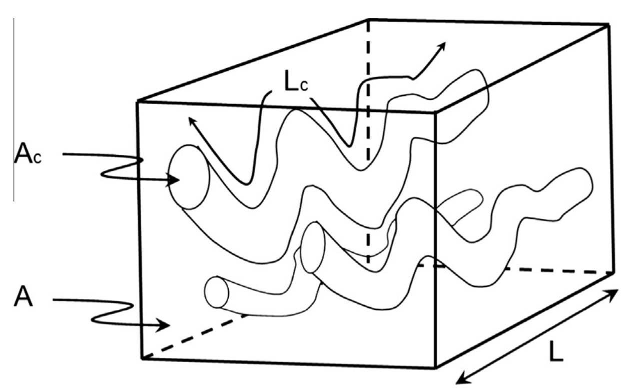

Fig. 1. A representative soil element with a bundle of capillary tubes that simulate the pore space (after Pfannkuch (1972)). The cross section area and length of the soil element are A and L, respectively. A_c and L_c are the cross section area and length of a single tube c, respectively. The radii of the bundle of capillary tubes are randomly distributed and the associated probability density function is $f(r)$.