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Mapping soil water retention curves via spatial Bayesian hierarchical models

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SUMMARY

Soil water retention curves are an important parameter in soil hydrological modeling. These curves are usually represented by the van Genuchten model. Two approaches have previously been taken to predict curves across a field – interpolation of field measurements followed by estimation of the van Genuchten model parameters, or estimation of the parameters according to field measurements followed by interpolation of the estimated parameters. Neither approach is ideal as, due to their two-stage nature, they fail to properly track uncertainty from one stage to the next. In this paper we address this shortcoming through a spatial Bayesian hierarchical model that fits the van Genuchten model and predicts the fields of hydraulic parameters of the van Genuchten model as well as fields of the corresponding soil water retention curves. This approach expands the van Genuchten model to a hierarchical modeling framework. In this framework, soil properties and physical or environmental factors can be treated as covariates to add into the van Genuchten model hierarchically. Consequently, the effects of covariates on the hydraulic parameters of the van Genuchten model can be identified. In addition, our approach takes advantage of Bayesian analysis to account for uncertainty and overcome the shortcomings of other existing methods. The code used to fit these models are available as an appendix to this paper. We apply this approach to data surveyed from part of the alluvial plain of the river Rhône near Yenne in Savoie, France. In this data analysis, we demonstrate how the inclusion of soil type or spatial effects can improve the van Genuchten model's predictions of soil water retention curves.

1. Introduction

Soil water retention curves are one of the most important parameters in soil hydrological modeling. These curves characterize water storage and pore distribution in soils and are an essential input to drive models that simulate soil water balance for climate and environmental monitoring as well as models of crop yield management. These curves are usually represented by an equation with several parameters that describe the relationship between water content and potential or pressure head. This parametric representation also allows the calculation of unsaturated hydraulic conductivity based on the assumed pore-distribution models (Mualem, 1976; Collis-George, 2014).

Various parametric equations have been proposed for modeling water retention curves, including Brooks and Corey (1964), van Genuchten (1980), and Kosugi (1996). Bimodal pore-distribution models such as Durner (1994) also have been proposed. The van Genuchten (VG) model is the most commonly used. Using the notation similar to Voltz and Goulard (1994), the model is written as

$$W(h) = \frac{W_s - W_r}{[1 + (\alpha h)^n]^m} + W_r, \quad W_s, W_r, \alpha, m > 0; \quad n > 1,$$

where $W(h)$ represents the water content (in gg^{-1}) at pressure head h (in m), W_s is the saturated water content (in gg^{-1}), W_r is the residual water content (in gg^{-1}), and α (in m^{-1}), n and m are shape parameters. The parameters W_s and W_r indicate the water content as $h \to 0$ and $h \to \infty$, respectively. The parameters α and n are related with the inverse of air entry suction and the pore-size distribution, respectively. Typically, since n is closely related to m, van Genuchten (1980) proposed replacing m with $1 - 1/n$. This special case of the VG model can thus be written as

$$W(h) = \frac{W_s - W_r}{[1 + (\alpha h)^n]^{1-\frac{1}{n}}} + W_r, \quad W_s, W_r, \alpha > 0; \quad n > 1. \tag{1}$$

In this setting, the number of parameters reduces to four. This form of the VG model has been widely used in addressing characteristic

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properties of soil water content. Our study also considers this form of the VG model.

Many methods have been proposed to estimate the hydraulic parameters of the VG model via measured water retention data. In general, they can be divided into two categories:

1. Use optimization techniques to minimize the sum of squared errors between the observed and modeled water retention. In this case, the Levenberg–Marquardt algorithm of nonlinear least-squares methods is used in the RETC program (van Genuchten et al., 1991) and the SWRC Fit (Seki, 2007). Other global optimization techniques have also been proposed such as a genetic algorithm (Vrugt et al., 2001) or simulated annealing (Younes et al., 2013).
2. Use Bayesian approaches in combination with Markov chain Monte Carlo (MCMC) (Abbaspour et al., 1997; Vrugt et al., 2003). The advantage of this approach is to provide the posterior distribution of parameters rather than a set of single values (i.e., point estimators).

Alternatively, pedotransfer functions (PTFs) (see Vila et al., 1999; Vereecken et al., 2010, and the references therein) have also been used to estimate the hydraulic parameters, but this is an indirect method for predicting parameters from other more easily measured soil properties.

Most research on the estimation of hydraulic parameters focuses on finding the best parametric model that can characterize soil hydraulic properties corresponding to water retention curves (Vrugt et al., 2003). Such research typically does not consider spatial variability of soil hydraulic properties, which can cause high variations in water transport processes. In addition, water retention curves are usually expensive to measure. Thus, spatial prediction of these properties is an important topic. Although researchers have been looking into efficient ways to predict water retention parameters across a field or landscape, determining and describing the spatial pattern of soil physical properties remains a difficult task for modeling landscape-scale soil–water processes (Wendroth et al., 2006). Voltz and Goulard (1994) proposed a two-stage approach to address this task. They first interpolated water content at different measured pressure heads and then used a least squares technique based on Marquardt's maximum neighborhood method to estimate VG parameters at each location of interest. Similarly, Saito et al. (2009) used SWRC Fit to estimate VG parameters at each location where data were measured, and conducted ordinary kriging for interpolation at locations in between. Furthermore, they evaluated two procedures: (1) fitting the VG model to the data and then interpolating the parameters, and (2) interpolating individual water retention measurements and then fitting the VG model at each interpolated location. Their results showed that the later procedure performed better when the mean absolute error of water content was used as the evaluation criteria. However, the approach of Voltz and Goulard (1994) and the latter one of Saito et al. (2009) took no advantage of the spatial variability of the VG model parameters to map water retention content curves. On the other hand, since these two approaches fit local VG models using a small sample size of measured water retention data, the estimated hydraulic parameters may be imprecise which can affect the precision of water retention curves. For example, the sample size of measured water retention at each location was eleven in the study of Saito et al. (2009). Consequently, abnormal observations would reduce the precision of the first procedure of Saito et al. (2009). Importantly, these approaches use least-squares based techniques to fit the VG model. As such, they lack the ability to account for uncertainty of the hydraulic parameters and could underestimate the uncertainty of predicted water retention curves.

In this paper, we propose a spatial Bayesian hierarchical approach to estimate and predict the VG model parameters and further, to predict their corresponding water retention curves. This approach allows soil properties and other physical or environmental factors to be incorporated into the VG model hierarchically to interpret and predict the variation of soil water retention curves. If *a priori* knowledge is known, PTFs can be included in this model as well. In some sense, this approach can be thought as a hierarchical VG model. It is important to note that although Abbaspour et al. (1997) and Vrugt et al. (2003) used Bayesian approaches to estimate the VG model parameters, their approaches did not consider spatial effects and cannot incorporate useful extraneous variables to improve the performance of the VG model. With our approach it is feasible to infer effects of covariates on hydraulic parameters corresponding to soil water retention curves. In addition, different from the two-stage approaches of Voltz and Goulard (1994) and Saito et al. (2009), our approach can estimate and predict the hydraulic parameters and water retention curves simultaneously using all data from the study area. Importantly, since our approach is in the Bayesian paradigm, the uncertainty of the hydraulic parameters and water retention curves can be quantified. A comprehensive introduction to the Bayesian paradigm, and the computational techniques within this paradigm, are beyond the scope of this paper. We cite related papers both from application journals and where necessary, from statistical ones. A good resource for interested readers is Carlin and Louis (2001) or Gelman et al. (2003).

We applied this approach to data that were previously surveyed from part of the alluvial plain of the river Rhône near Yenne in Savoie, France. Our example demonstrates how the inclusion of covariate information such as soil type or the inclusion of spatial effects in the model can lead to improvements in the performance of the VG model in predicting water retention curves. The fact that this is achieved while simultaneously accounting for uncertainty both in the VG model and in the spatial interpolation, marks an important contribution to this field of research.

2. Data and method

2.1. Data description

Voltz and Goulard (1994) surveyed water retention from part of the alluvial plain of the river Rhône near Yenne in Savoie, France. Fig. 1 illustrates the spatial distribution of 75 sites in the study area from two sampling schemes: 54 are from a rectangular grid with points equally spaced at 100 and 200 m intervals in the x and y direction, respectively, and 21 are from a square grid with points equally spaced at 141 m intervals. Because of gravel content, Fig. 1 illustrates that the two sampling scheme were not sampled completely. At each location, undisturbed topsoil aggregates were collected at a depth of 40 cm. Their gravimetric water contents were measured at eight levels of pressure head: -0.1, -0.5, -1, -2, -4, -9, -30, and -150 m in a pressure plate extractor.

The study area contained six soil types which differ in terms of their soil texture and drainage characteristics. Voltz and Goulard (1994) described these six classes as follows: silt loam over loam (type 1), homogeneous silt loam (type 2), silty clay loam over poorly drained silty clay with marked gleyic features in depth (type 3), homogeneous silt loam with shallow phreatic water (type 4), loam over gravelly sand (type 5), and loam with angular gravel and presence of shallow phreatic water (type 6). However, the 75 sampled locations only covered five of these soil types since sites of type 6 could not be sampled. Fig. 1 illustrates the spatial distribution of five soil classes across the 75 sampled locations. Fig. 2(a) demonstrates the observed water retention curves and associated mean curve as a function of pressure head. The red curve highlighted in