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Urban flood modeling with porous shallow-water equations: A case study of model errors in the presence of anisotropic porosity

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ABSTRACT

Porous shallow-water models (porosity models) simulate urban flood flows orders of magnitude faster than classical shallow-water models due to a relatively coarse grid and large time step, enabling flood hazard mapping over far greater spatial extents than is possible with classical shallow-water models. Here the errors of both isotropic and anisotropic porosity models are examined in the presence of anisotropic porosity, i.e., unevenly spaced obstacles in the cross-flow and along-flow directions, which is common in practical applications. We show that porosity models are affected by three types of errors: (a) structural model error associated with limitations of the shallow-water equations, (b) scale errors associated with use of a relatively coarse grid, and (c) porosity model errors associated with the formulation of the porosity equations to account for sub-grid scale obstructions. Results from a unique laboratory test case with strong anisotropy indicate that porosity model errors are smaller than structural model errors, and that porosity model errors in both depth and velocity are substantially smaller for anisotropic versus isotropic porosity models. Test case results also show that the anisotropic porosity model is equally accurate as classical shallow-water models when compared directly to gage measurements, while the isotropic model is less accurate. Further, results show the anisotropic porosity model resolves flow variability at smaller spatial scales than the isotropic model because the latter is restricted by the assumption of a Representative Elemental Volume (REV) which is considerably larger than the size of obstructions. These results point to anisotropic porosity models as being well-suited to whole-city urban flood prediction, but also reveal that point-scale flow attributes relevant to flood risk such as localized wakes and wave reflections from flow obstructions may not be resolved.

1. Introduction

Urban flood modeling is now possible at centimetric resolution or better with modern laser scanning data and flood models (Bates, 2012; Sampson et al., 2012), but it is not advisable at this resolution over entire floodplains as the computational costs and memory demands are forbidding except on massively parallel computing architectures. Commonly used models are constrained by the Courant, Friedrichs, Lewy (CFL) condition for both stability and accuracy which dictates nearly an order-of-magnitude increase in computational effort every time the mesh resolution is doubled. For a Cartesian grid with a cell size of Δx, the computational cost C of integrating a flood over a specified duration will scale as the product of the required number of computational cells n_c and time steps n_t,

$$C \sim n_c n_t \sim \frac{1}{\Delta x^3} \qquad (1)$$

because $n_c \sim \Delta x^{-2}$ and the CFL requirement to scale Δt with Δx. Thus, halving the cell size causes an eight fold increase in computational effort (nearly an order of magnitude) and at least a fourfold increase in memory demands.

Porous shallow-water equations (porosity models) resolve urban flooding at a relatively coarse (and efficient) resolution compared to available geospatial data using additional parameters that account for sub-grid scale topographic features affecting the movement and storage flood water (Defina, 2000; Yu and Lane, 2005; McMillan and Brasington, 2007; Sanders et al., 2008; Soares-Frazão et al., 2008; Cea and Vázquez-Cendón, 2010; Chen et al., 2012; Guinot, 2012; Schubert and Sanders, 2012). In practice, the

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idea is to use a cell size on the order of meters or dekameters. This gives rise to models that resolve flooding at the *pore scale* roughly corresponding to the width of roadways and open spaces between buildings, in contrast with classical shallow-water models that resolve flooding at the *point scale*, as approximated by the grid resolution. Importantly, porosity models enable massive reductions in computational effort compared with classical shallow-water models as a result of the scale difference.

Sanders et al. (2008) and Guinot (2012) introduce two alternative formulations of porosity models to capture porosity anisotropy, which can be expected in practical applications. Anisotropy occurs in urban landscapes when there are preferential flow directions such as wide streets and narrow alleys aligned in perpendicular directions. Hypothetical examples of anisotropic flow have been presented in previous studies (Sanders et al., 2008; Guinot, 2012), including numerous cases with angled channel-like flows through urban areas. Additionally, Schubert and Sanders (2012) present a field-scale application of an anisotropic porosity model that outperforms models based on the classical shallow-water equations.

Porosity heterogeneity exists when the size of flow paths is spatially variable, and different porosity models resolve heterogeneity over different scales. Isotropic porosity models are restricted to scales larger than the length scale of the Representative Elemental Volume (REV). This is typically an order of magnitude larger than the scale of flow obstructions in urban flood applications, nominally a kilometer or more (Guinot, 2012). On the other hand, the anisotropic porosity model developed by Sanders et al. (2008) does not require the existence of an REV and can resolve heterogeneity at the grid scale.

Since porosity anisotropy is a critical consideration for practical applications, this study presents modeling of a unique experimental test case involving dam-break flow through an anisotropic array of obstructions, which builds on earlier experimental work and modeling studies focused on isotropic arrays of obstructions (Testa et al., 2007; Soares-Frazão and Zech, 2008). A classical shallow-water model and both isotropic and anisotropic porosity models are applied and calibrated. The objective is to measure and report the magnitude of porosity model errors in an absolute sense and also relative to other errors which collectively limit the overall accuracy of the model. A better understanding of errors is needed to effectively use porosity models in flood hazard mapping. Three types of errors are reported: (a) structural model errors associated with the shallow-water equations which constitute the foundation of the porosity models, (b) scale errors arising from a grid size that matches the pore scale instead of the point scale, and (c) porosity model errors associated the parameterization of sub-grid scale obstructions. Results point to significant differences in porosity model errors between porosity model formulations.

2. Methods and materials

2.1. Porosity definition

Porosity can be defined in more than one way, namely as a volume average fraction of pore space in a porous media or as an areal average fraction of pore space, as in a slice through the porous medium (Bear, 1988). Both volumetric and areal porosity can be expected to vary spatially in the case of a heterogeneous porous medium, and areal porosity can also vary with the orientation of the plane over which the areal average is taken, and thus exhibit anisotropy. If an urban land surface filled with solid features is taken as a porous medium, then the pore space represents the gaps between the solid features, the volumetric porosity represents the fraction of the land surface able to store water, and the areal porosity represents the fraction of space available for flood conveyance which is directionally dependent.

2.2. Porous shallow-water equations

The anisotropic porosity model of Sanders et al. (2008) is written as integral statements of mass and momentum conservation for an arbitrary 2D domain Ω with boundary Γ and unit outward normal vector $\mathbf{n}$ as follows,

$$\frac{\partial}{\partial t}\int_{\Omega} i\mathbf{U}\,d\Omega + \oint_{\Gamma} i\mathbf{E}\cdot\mathbf{n}\,d\Gamma = \oint_{\Gamma} i\mathbf{H}\cdot\mathbf{n}\,d\Gamma + \int_{\Omega} i\mathbf{S}\,d\Omega \tag{2}$$

where

$$\mathbf{U} = \begin{pmatrix} h \\ uh \\ vh \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} uh & vh \\ u^2h + \frac{1}{2}gh^2 & uvh \\ uvh & v^2h + \frac{1}{2}gh^2 \end{pmatrix} \tag{3}$$

$$\mathbf{S} = \begin{pmatrix} 0 \\ -(c_D^f + c_D^b)uV \\ -(c_D^f + c_D^b)vV \end{pmatrix} \quad \mathbf{H} = \begin{pmatrix} 0 & 0 \\ \frac{1}{2}gh|_{\eta_o}^2 & 0 \\ 0 & \frac{1}{2}gh|_{\eta_o}^2 \end{pmatrix} \tag{4}$$

where u = x-component of velocity, v = y-component of velocity, g = gravitational constant, $V = (u^2 + v^2)^{1/2}$, c_D^f is a ground friction drag coefficient, c_D^b is a drag coefficient for sub-grid scale flow obstructions, and $h|_{\eta_o}$ is the depth corresponding to a piecewise constant water surface elevation η_o and piecewise linear ground elevation z within Ω. The $\mathbf{H}$ term is introduced to transform the classical ground slope source term to a boundary integral that preserves stationary solutions. Based on the limits of this transformation, the momentum equations appearing in Eq. (2) are restricted to numerical schemes that are first- or second order accurate in space (Sanders et al., 2008).

The variable $i(x, y)$ appearing in Eq. (2) is defined for the spatial domain $D \in \mathbf{R}^2$ and represents a binary density function that takes on a value of zero or unity depending on the presence or absence of a solid flow barrier as follows (Sanders et al., 2008),

$$i(x,y) = \begin{cases} 0 & \text{if} \quad (x,y) \in D_b \\ 1 & \text{otherwise} \end{cases} \tag{5}$$

where D_b is a subdomain of D that corresponds to solid obstacles. Two grid-based porosity parameters are dependent on the density function (Eq. (5)) as follows,

$$\phi_j = \frac{1}{\Omega_j}\int_{\Omega_j} i\,d\Omega \qquad \psi_k = \frac{1}{\Gamma_k}\int_{\Gamma_k} i\,d\Gamma \tag{6}$$

where Ω_j corresponds to the two-dimensional (2D) spatial domain of the jth computational cell and Γ_k corresponds to the kth computational edge of a mesh. Note that ϕ_j represents the fraction of a cell area occupied by voids, and ψ_k represents the fraction of a cell edge occupied by voids. Consequently, these parameters affect the relative storage of cells and conveyance between cells, respectively. Importantly, anisotropic blockage effects are explicitly resolved by the distribution of ψ_k values across the computational mesh. It is noted that isotropic porous shallow-water equations can be recovered from Eq. (2) under the assumption that $\phi_j = \psi_k \forall k$. Additionally, Eq. (2) reverts to the classical shallow-water equations in the limit that $i(x, y) = 1$.

Presently it is not clear how well isotropic and anisotropic porosity models resolve flow at the pore scale where information is needed to assess the risks facing individual land parcels in an urban area, especially when the obstructions exhibit anisotropy. Eq. (2) resolves flow properties on a grid-cell by grid-cell basis which corresponds to the pore scale since the model requires a grid