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Axisymmetric gravity currents within porous media: First order solution and experimental validation

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SUMMARY

Spreading of gravity currents in porous media has traditionally been investigated analytically by means of similarity solutions under the Dupuit–Forchheimer approach. We present a novel formulation to analyse the axisymmetric propagation of single-phase gravity currents induced by the release of a time-variable volume of fluid in a porous domain. Our approach is based on a first order expansion of the velocity potential that allows for the presence of vertical Darcy velocities. Coupling the flow law with mass balance equations leads to a PDE which admits a self-similar solution for the special case in which the volume of the fluid fed to the current increases at a rate proportional to t^3. A numerical solution is developed for rate proportional to t^α with $\alpha \neq 3$. Current profiles obtained with the first order solution have a finite height at the origin. Theoretical results are compared with two experimental datasets, one having freshwater and the other air as an ambient fluid. In general, experimental current profiles collapse well onto the numerical results; the first order solution shows a marked improvement over the zeroth order solution in interpreting the current behaviour near the injection point. A sensitivity and uncertainty analysis is conducted on both the first order and zeroth order theoretical model. The sensitivity analysis indicates that the flow process is more sensitive to porosity variations than to other parameters. The uncertainty analysis of the present experimental data indicates that the diameter of glass beads in an artificial porous medium is the source of most of the overall uncertainty in the current profile.

1. Introduction

Flows in porous media driven by the force of gravity acting on density differences between an intruding and an ambient fluid are usually termed gravity currents, and are frequent in many environmental and industrial processes. Notable examples include injection of gas or liquid into natural reservoirs to improve recovery of oil and heat, release of agents or environmental carriers into aquifers for remediation of groundwater contamination, carbon dioxide sequestration in deep formations to reduce greenhouse gas emissions, and saltwater intrusion in coastal aquifers. These important applications, with far-reaching economic implications, have prompted in the past three decades the development of significant research advances, both of theoretical and experimental nature, on the propagation of gravity currents in porous media. Among these, a number of closed-form approaches were developed to analyse one-phase flows in a variety of geometric settings and with different boundary conditions. Noteworthy examples include flow over an horizontal surface in plane (Huppert and Woods, 1995) and radial geometry (Lyle et al., 2005) generated by an instantaneous or continuous release of fluid. These solutions were extended to incorporate two-layer flow (Woods and Mason, 2000), the effect of a sloping bottom (Vella and Huppert, 2006; Koussis et al., 2012), the action of impermeable confining boundaries (Golding and Huppert, 2010), and drainage effects (Pritchard et al., 2001). In other cases, the volume of fluid injected in the origin of a semi-infinite porous domain is not conserved, as the fluid mound partially drains back; the corresponding initial value problem for an instantaneous injection is known as the dipole, as the first spatial moment of the mound is conserved. The closed-form solution to this problem, originally derived by Barenblatt and Zel'dovich (1957), was extended by King and Woods (2003) to include drainage and by Mathunjwa and Hogg (2007) to incorporate a vertical variation in permeability.

All of these studies rely on a thin-current assumption, in which the component of Darcy flow perpendicular to the main direction

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of motion is neglected, applying the Dupuit–Forchheimer approximation. This entails a logarithmic singularity at the origin of a radial coordinate system, as clearly shown by Dussan and Auzerais (1993) and further discussed by Li et al. (2005) and Di Federico et al. (2012).

Several different approaches were developed to remove this approximation and describe the flow field more accurately. Dagan (1967) derived a second order approximate theory for steady free surface 2-D flow in porous media via matched asymptotic expansions, demonstrating that the zero order term of the expansion coincides with the Dupuit–Forchheimer approximation. He adopted a general shallow-flow expansion introducing a small parameter to stretch differently the horizontal and the vertical coordinates, then assumed an expansion of the potential involving powers of the square of the small parameter; the series included only even powers to match the inner and the outer expansion. An extension to axisymmetric geometry to deal with pumping wells was developed with a similar mathematical technique by Dagan (1968). The modelling of groundwater periodic motion in aquifers, as generated by tidal excursions, often requires an extension of the zeroth order theory in order to include the effects of vertical velocity and remove the hydrostatic hypothesis. To this purpose, a shallow water expansion was adopted in Parlange et al. (1984) to evaluate a hierarchy of functions representing the terms of a series development of the hydraulic head.

An asymptotic expansion in the vertical-to-horizontal aspect ratio was adopted by Yortsos (1995), who also included the effects of different permeabilities in the vertical and horizontal direction. He demonstrated rigorously that the Dupuit approximation (and several other approximations in different contexts) is obtained considering only the leading order term of the expansion.

A step-wise approach, where the zeroth order term corresponds to the Dupuit–Forchheimer approximation, was adopted in Nielsen et al. (1997) to model tidal water table waves. They introduced the effects of a vertical velocity varying linearly in the vertical on the pressure field. A similar approach is also reported in Knight (2005) for several test-case problems.

In a similar way, Nordbotten and Celia (2006) developed a scheme to correct the vertical equilibrium model (i.e., Dupuit approximation) by introducing a vertical velocity with a linear variation, that was observed in numerous numerical simulation; as a consequence, the pressure variation in the vertical is quadratic. de Loubens and Ramakrishnan (2011) gave a formal justification of the vertical equilibrium approach through a perturbation theory, without making assumptions on the pressure distribution.

Here, we adopt a series expansion of the velocity potential which is polynomial in the vertical coordinate and contains only even terms as a consequence of the no-flow boundary condition at the horizontal bottom. The asymptotic expansion is in power of a small parameter equal to the ratio between vertical and horizontal length scales; at the zeroth order, it reduces to the Dupuit approximation, like the approaches of other researchers. All the terms in the series are expressed iteratively as space derivatives of the zeroth order potential. While other approaches (e.g., Dagan, 1967) adopt a series with only even powers, to our knowledge the derivation of the higher order terms as a function of the zeroth order term is novel in the field of flow in porous media. This relationship is an advantage since it allows the direct implementation of the effects at every order without a hierarchical approach, albeit with an increase in the order of the differential problem describing the physical processes.

The aforementioned approach is used to derive a first order correction to the similarity solution developed by Lyle et al. (2005) to analyse the spreading of a gravity current in a porous medium in radial geometry. The solution thereof was successfully used to interpret accumulation of supercritical carbon dioxide beneath low-permeability mudstone layers at the Sleipner site in the North Sea (Bickle et al., 2007), following injection of CO_2 at an approximately constant rate since 1996. An extension of their formulation allowing for nonzero vertical components of the velocity field may be of interest also in view of recent further refinements of the model incorporating two-phase flow (Golding et al., 2013). The present approach can be of interest also in all flows characterised by relevant gradient of the interface and possibly in non stationary conditions, such as coastal salt water intrusion under the effect of tidal fluctuations.

The paper is organised as follows. Section 2 presents the theoretical formulation of the problem, while Section 3 illustrates the results, obtained with self-similar transformation in the special case $\alpha = 3$ and via numerical integration in the general case. In Sections 4 and 5, theoretical results are compared with laboratory experiments. Section 4 describes the experimental apparatus used by Longo et al. (2013) to analyse the spreading of gravity currents in porous media, and illustrates the comparison with their experiments. Throughout the analysis, the sources of uncertainty in the modelling approach and their respective weights are highlighted. A systematic comparison with results from a second experimental dataset by Lyle et al. (2005) is presented in Section 5 following the same approach. A more extensive comparison of the theoretical model with experimental results is available online as Supplementary Material.

A set of conclusions (Section 6) closes the paper, while the Appendix includes further details on the model sensitivity analysis.

2. Formulation of the problem

Consider an axisymmetric gravity current of a Newtonian liquid of density ρ propagating above an horizontal impermeable boundary and intruding in an infinite porous domain of depth h_0, saturated with an ambient fluid of density $\rho - \Delta\rho$ (Fig. 1).

The current is released into the porous medium by a line source along the vertical axis, extends from the origin to a coordinate denoted by $r_N(t)$, and has a volume given by Qt^{α}, where Q and α are positive constants, and $\alpha = 0$ and $\alpha = 1$ indicate respectively an instantaneous release of a fixed volume and a constant volume influx; the actual discharge for $\alpha \neq 0$ is given by $Q_d = \alpha Q t^{\alpha-1}$. The equation of mass conservation is

$$\nabla \cdot \mathbf{u} = 0, \tag{1}$$

with $\mathbf{u} = (u, w)$ representing Darcy velocity, given by Darcy's law (Bear, 1988; Phillips, 1991; Dullien, 1992):

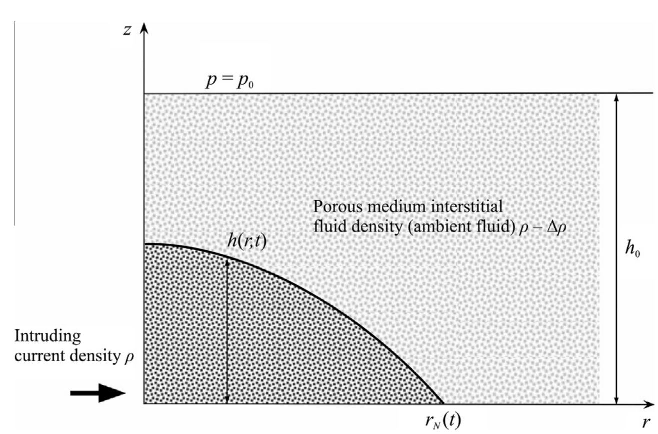

Fig. 1. Layout of the problem.