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Multi-scale approach to invasion percolation of rock fracture networks

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SUMMARY

A multi-scale scheme for the invasion percolation of rock fracture networks with heterogeneous fracture aperture fields is proposed. Inside fractures, fluid transport is calculated on the finest scale and found to be localized in channels as a consequence of the aperture field. The channel network is characterized and reduced to a vectorized artificial channel network (ACN). Different realizations of ACNs are used to systematically calculate efficient apertures for fluid transport inside differently sized fractures as well as fracture intersection and entry properties. Typical situations in fracture networks are parameterized by fracture inclination, flow path length along the fracture and intersection lengths in the entrance and outlet zones of fractures. Using these scaling relations obtained from the finer scales, we simulate the invasion process of immiscible fluids into saturated discrete fracture networks, which were studied in previous works.

1. Introduction

Transport of fluids in fractured geological media plays an important role in different applications such as subsurface hydrology, hydrocarbon recovery from natural reservoirs, safe storage facilities for captured CO_2 or hazardous wastes (Adler et al., 2013; Sahimi, 1995). The prediction of the penetration of an immiscible fluid into a fully saturated fracture network and its percolation is among the most challenging topics in sub-surface hydrology. Fractured rock, instead of behaving like an equivalent continuum, is only sparsely fractured, restricting fluid flow to a small part of the connected fractures (Reeves et al., 2008). Fluids can take multiple pathways, can be trapped, and exhibit path instabilities and scale dependencies, what makes the prediction of effective properties rather challenging. Detailed descriptions however are rarely capable of capturing more than a handful of interconnected fractures and fail in representing the complex system dynamics emerging from the interaction of a huge number of locally interacting connected fractures. A partial solution to this dilemma was found in the Discrete Fracture Network (DFN) approach that assumes fluid flow to be entirely localized in the network of connected fractures (Smith and Schwartz, 1984; Huseby et al., 2001). DFN models have been widely applied to study fluid flow and transport characteristics of several fractured rocks with low permeability and fracture density (Reeves et al., 2008; Sisavath et al., 2004; Renshaw, 1999). As one expects, transport properties of the networks are strongly affected by the fracture density, size and interactions (Khamforoush and Shams, 2007; Koudina et al., 1998; Dreuzy et al., 2000). For a detailed description of the physics of fracture networks we refer to the reviews by Berkowitz (2002), Sahimi (1995), and Adler et al. (2013).

Since fluid flow in a single fracture is at the bottom of any DFN model, a detailed description of its geometry and hydraulic behavior is essential (Charmet et al., 1990; Pompe et al., 1990; Adler et al., 2013). Realistic topological measurements of natural fracture planes revealed spatial correlations (Candela et al., 2012). From those, aperture field distributions can be obtained (Kumar et al., 1997) and applied for studying the hydraulic behavior of the single fracture (Konzuk and Kueper, 2004). The most striking observation is that significant parts of the natural fractures have zero aperture since they are contact zones, while the fluid flow is naturally localized in areas with non-zero aperture which are also called open zones. Hence, the fluctuations in the aperture field lead to fluid flow in a network of channels (Katsumi et al., 2009) and fracture interactions are governed by the characteristics of these two networks. To model the displacement of immiscible fluids inside DFN models, invasion percolation (IP) theory proved to be an efficient tool (Wilkinson and Willemsen, 1983). In fact, IP is a modified

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form of ordinary percolation (Broadbent and Hammersley, 1957), only with a well-defined sequence of invasion events. In IP theory both gravity and viscous effects are neglected and only capillary forces are considered. However, IP is a valid approximation to describe the slow immiscible displacement of two phase flow in fracture networks.

In this work, details of the single fracture with its inherent disorder are upscaled to the fracture network by a two-step coarse graining method. Vectorized artificial channel networks (ACN) are used, that are highly compressed representations of channelized flow on the single fracture scale obtained from Finite Element (FE) simulations with heterogeneous aperture fields. ACNs are used to calculate the scaling behavior of hydraulic transport properties of the single fractures in terms of size dependent equivalent apertures, as well as size and angle dependent fracture–fracture interactions for entry apertures. The numerically obtained rules are then incorporated into a fracture-network consisting of 4000 intersecting fractures for a modified invasion percolation (MIP) simulation with two-phase flow (Wettstein et al., 2012). This way we obtain a more realistic physical description of the invasion process with coarse grained information from the fracture scale.

2. Methodology

In this methodological section, first flow at the fracture scale is addressed by FE simulations and as prerequisite for data compression for the ACN. As a next step we describe fracture interaction parameters, effective properties and their consideration on the DFN scale in trapping MIP.

2.1. Flow at the fracture scale

Single fractures have fundamental meaning in fracture network modeling (Cappa, 2011) since from this, scale size effects emerge. The effect of fracture aperture and roughness on hydraulic properties was already addressed from the theoretical and numerical perspectives in the past (Drazer and Koplik, 2000, 2002; Auradou et al., 2001; Berkowitz, 2002; Sahimi, 1995). In general, fracture surfaces exhibit spatial correlation (Brown et al., 1986; Méheust and Schmittbuhl, 2001) and are self-affine with roughness (Hurst) exponent close to 0.8 for diverse materials (Bouchaud et al., 1990; Ansari-Rad et al., 2012; Dyer et al., 2012). However also roughness exponents close to 0.6 can be observed in the direction of slip at laboratory scale samples (Amitrano and Schmittbuhl, 2002) and even at the scale of natural faults (Bistacchi et al., 2011; Candela et al., 2012). In principle, fractures with a constant fracture opening have constant aperture and could be reduced to two parallel plates with Hagen-Poiseuille flow as well as the cubic law (Zimmerman and Bodvarsson, 1996; Konzuk and Kueper, 2004). However huge fluctuations of the aperture field arise due to contact zones between two facing fracture surfaces (Brown, 1987; Dreuzy et al., 2012; Katsumi et al., 2009). Detailed explanations of self-affine fracture surfaces and aperture field distributions can be found for example in Adler et al. (2013) and Brown (1995). Isotropic self-affine fracture surfaces are constructed using a spectral method based on fractional Brownian motion (Schrenk et al., 2013). In a consecutive step a copy of the surface is vertically and horizontally displaced with respect to the original, resembling homogeneous shear displacement. The local aperture is then taken as distance between surfaces while overlapping and contacting regions are assigned zero apertures (Auradou et al., 2005; Durham, 1997; Katsumi et al., 2009).

In this study, uncorrelated random disordered fracture surfaces and hence uncorrelated random disordered aperture fields are used for generality, since there is no focus on a peculiar geological formation. However any type of aperture field can be treated analogously. The utilized disorder is given by a power-law probability density distribution such that the aperture h_i of each site i can be generated as $h_i = \exp(B(x_i - 1))$ with the disorder parameter B and x_i being a uniformly distributed random number in $[0, 1]$ (Oliveira et al., 2011; Braunstein et al., 2002). For each realization, an aperture h_i, $i = 1, \ldots, L^d$, (where L is the size of the aperture field, e.g. of a square lattice and d is the dimension of the system) is assigned to each site according to the mentioned power-law distribution. The number of cells is chosen with respect to the characteristic length l for the aperture field that is assumed to be unit length. Between neighboring sites, values are interpolated by bi-cubic interpolation (see Fig. 1(a)).

To model fluid flow inside of single fractures, the lubrication approximation, hence Stokes flow for small Reynolds number ($Re < 1$), as well as negligible gravity and inertial forces are assumed. The Stokes equation between two parallel plates relates the pressure field p to the velocity field υ by

$$\nabla p = \mu \Delta \upsilon \tag{1}$$

with the dynamic viscosity μ of fluid. For a parallel plate the velocity υ only depends on plate distance (z-direction) while the pressure p is only dependent on the x-direction. Hence Eq. (1) can be rewritten as

$$\frac{dp}{dx} = \mu \frac{d^2 \upsilon}{dz^2} = C, \tag{2}$$

where C is a constant (Méheust and Schmittbuhl, 2001). By solving Eq. (2), one gets the well-known "cubic law", which gives a good estimate of the volumetric flow rate q through a single fracture as a function of the pressure gradient ∇p in the flow direction. A cubic law equation is given by

$$q = W \frac{h^3}{12 \mu L} \nabla p, \tag{3}$$

where h is the plate distance or fracture aperture and W denotes the width of the fracture perpendicular to the flow direction (see Fig. 1(a)).

To simulate fluid flow through a rough fracture, an aperture field has to be used. Using the lubrication approximation, pressure p only depends on the x- and y-direction, leading to the following form of the Stokes equation:

$$\nabla p(x, y) = \mu \frac{d^2 \upsilon(x, y, z)}{dz^2}. \tag{4}$$

In order to simplify Eq. (4), the local direction $\hat{u}(x, y)$ and the local coordinate u along this direction for the pressure gradient are introduced by

$$\frac{dp}{d\hat{u}} = \mu \frac{d^2 \upsilon(x, y, z)}{dz^2}. \tag{5}$$

In order to reduce the problem to a two dimensional system, the local flow rate direction $\hat{q}(x, y)$ can be obtained by integration along the z direction

$$\hat{q}(x, y) = \int_{z_l}^{z_u} \upsilon(x, y, z) dz = \frac{\hat{h}(x, y)^3}{12 \mu} \nabla p, \tag{6}$$

with z_l and z_u being the lower and upper surface heights respectively, hence the local cubic law with the aperture field $\hat{h}(x, y)$. For this two-dimensional approach, the main flow direction is assumed to be the x-direction with constant pressure at the inlet p_{in} and outlet p_{out}. Also no-flow conditions are defined on boundaries parallel to the flow direction. Note that all quantities are kept in a dimensionless form by using the characteristic units of the numerical