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A new approach to model weakly nonhydrostatic shallow water flows in open channels with smoothed particle hydrodynamics

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SUMMARY

A new approach to model weakly nonhydrostatic shallow water flows in open channels is proposed by using a Lagrangian meshless method, smoothed particle hydrodynamics (SPH). The Lagrangian form of the Boussinesq equations is solved through SPH to merge the local and convective derivatives as the material derivative. In the numerical SPH procedure, the present study uses a predictor–corrector method, in which the pure space derivative terms (the hydrostatic and source terms) are explicitly solved and the mixed space and time derivatives term (the material term of B_1 and B_2) is computed with an implicit scheme. It is thus a convenient tool in the processes of the space discretization compared to other Eulerian approaches. Four typical benchmark problems in weakly nonhydrostatic shallow water flows, including solitary wave propagation, nonlinear interaction of two solitary waves, dambreak flow propagation, and undular bore development, are selected to employ model validation under the closed and open boundary conditions. Numerical results are compared with the analytical solutions or published laboratory and numerical results. It is found that the proposed approach is capable of resolving weakly nonhydrostatic shallow water flows. Thus, the proposed SPH approach can supplement the lack of the SPH–Boussinesq researches in the literatures, and provide an alternative to model weakly nonhydrostatic shallow water flows in open channels.

1. Introduction

Due to the requirement of less computational cost compared to the Navier–Stokes equations, the shallow water equations are the most common choice to mathematically describe many hydraulic engineering problems in rivers and floodplains (Cunge et al., 1980; Morris, 2000; Chaudhry, 2008). The shallow water equations assume that the water pressure is only dependent on the total flow depth, resulting in a hydrostatic pressure distribution over the flow domain and the vertical motion is small enough to be neglected (Cunge et al., 1980; Chaudhry, 2008). So far many numerical researches (Morris, 2000) have reported that the shallow water equations are reasonably suitable for the representation of gradually varied flows in rivers and floodplains, however, they fail to accurately represent rapidly varied flows. For such violent flows, the ratio of the vertical-to-horizontal scales of motion is no longer small and the vertical acceleration significantly creates a nonhydrostatic pressure distribution that should be incorporated into the equations used to route these flows.

An appropriate option for the computations of rapidly varied flows is the Boussinesq equations, which are the depth-averaged version by utilizing the Boussinesq assumption (Chaudhry, 2008). They are the simplest class of mathematical models that expand the shallow water equations with the Boussinesq terms to capture weakly nonhydrostatic physics such as wave refraction and diffraction. The simulated outcomes based on the Boussinesq equations have been demonstrated to provide good predictions for a range of physical configurations such as dambreak flow transport (Mohapatra and Chaudhry, 2004), undular bore evolution (Favre, 1935; Peregrine, 1966), and solitary wave propagation (Devkota and Imberger, 2009). As a result, the Boussinesq equations have been prevailing in simulating weakly nonhydrostatic shallow water flows. In this study, the Boussinesq equations are adopted herein to remove the restriction of the hydrostatic assumption in the shallow water equations. The one-dimensional (1D) Boussinesq equations of continuity and momentum in an Eulerian form can be written as (Chaudhry, 2008).

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$$\underbrace{\frac{\partial h}{\partial t}}_{\text{Local}} + \underbrace{u\frac{\partial h}{\partial x}}_{\text{Convective}} + h\frac{\partial u}{\partial x} = 0 \tag{1}$$

$$\underbrace{\frac{\partial u}{\partial t}}_{\text{Local}} + \underbrace{u\frac{\partial u}{\partial x}}_{\text{Convective}} = \underbrace{-g\frac{\partial h}{\partial x}}_{\text{Hydrostatic pressure term}} + \underbrace{g(S_0 - S_f)}_{\text{Source term}} + \underbrace{\frac{h^2}{3}\left[\underbrace{\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right)}_{B_1} + \underbrace{u\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}\right)}_{B_2} - \underbrace{\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)^2}_{B_3}\right]}_{\text{Boussinesq terms}} \tag{2}$$

where t is the time, x is the space Cartesian coordinate, u is the depth-averaged velocity in the x-direction, g is the gravitational acceleration, h is the flow depth, S_0 is the bed slope, and $S_f(= n^2u|u|/h^{4/3})$ is the friction slope with n is the Manning roughness coefficient reflecting the roughness of the bottom. Three main terms (the hydrostatic, source and Boussinesq terms) are included in the right-hand side of Eq. (2). The Boussinesq terms, B_1, B_2, and B_3 account for the effect of vertical acceleration. The physical meaning of B_1 is the local acceleration in the vertical direction (z-direction), and B_2 and B_3 represent the convective acceleration in the x- and z-direction, respectively. For weakly nonhydrostatic shallow-water flows, the material term of B_1 and B_2 has major contributions compared to the third-derivative space term of B_3 (Mohapatra and Chaudhry, 2004). Therefore, it is sufficient to only include the material term of B_1 and B_2 in Eq. (2) as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -g\frac{\partial h}{\partial x} + g(S_0 - S_f) + \frac{h^2}{3}\left[\frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial x^2}\right) + u\frac{\partial}{\partial x}\left(\frac{\partial^2 u}{\partial x^2}\right)\right] \tag{3}$$

Conventionally, there are a variety of numerical methods that can be used to solve the Boussinesq equations, including finite difference methods (Zijlema et al., 2011), finite element methods (Walters, 2005) and finite volume methods (Denlinger and O'Connell, 2008). These Eulerian-based approaches can expend considerable efforts to yield satisfactory discretization and to reduce truncation errors for the nonlinear convective term using second-order numerical schemes. Similarly, it is also necessary to employ third- or higher-order accurate schemes to solve the third-order Boussinesq terms (Abbott, 1979; Basco, 1989; Chaudhry, 2008). Although such higher-order schemes have effective computations for weakly nonhydrostatic pressure correction, they are computational tediously and may still suffer from the challenge of grid-resolution problems (Mohapatra and Chaudhry, 2004; Devkota and Imberger, 2009). On the other side, some meshless methods have been applied to hydrodynamics recently. Among them, smoothed particle hydrodynamics (SPH) has been proved to have some numerical advantages (Wang and Shen, 1999; Ata and Soulaimani, 2005; Rodriguez-Paz and Bonet, 2005; De Leffe et al., 2010; Vacondio et al., 2011; Chang et al., 2011; Kao and Chang, 2012; Vacondio et al., 2012a,b; Chang and Chang, 2013), which are adequate to be used in solving the Boussinesq equations. Firstly, SPH is a particle method with Lagrangian nature. Particles move with the flow, and the convective term is merged into the material derivative so that the numerical dispersion error resulting from the convective term can be directly eliminated (Devkota and Imberger, 2009). The Lagrangian form of the 1D Boussinesq equations of continuity and momentum can be rewritten from Eqs. (1) and (3) as

$$\underbrace{\frac{\mathrm{D}h}{\mathrm{D}t}}_{\text{Material}} = -h\frac{\partial u}{\partial x} \tag{4}$$

$$\underbrace{\frac{\mathrm{D}u}{\mathrm{D}t}}_{\text{Material}} = \underbrace{-g\frac{\partial h}{\partial x}}_{\text{Hydrostatic pressure term}} + \underbrace{g(S_0 - S_f)}_{\text{Source term}} + \underbrace{\frac{h^2}{3}\frac{\mathrm{D}}{\mathrm{D}t}\left(\frac{\partial^2 u}{\partial x^2}\right)}_{\text{Material term of } B_1 \text{ and } B_2} \tag{5}$$

where $\frac{\mathrm{D}}{\mathrm{D}t}\left(= \frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)$ represents the material derivative. Obviously, in Eqs. (4) and (5), the Lagrangian description merges the local and convective derivatives as the material derivative.

In addition, SPH allows water particles freely moving in the computational domain without being confined in a fixed mesh. Consequently, SPH can conserve mass exactly and has strong capability to deal with large deformation problems (Liu and Liu, 2003). Therefore, it is a suitable numerical tool to solve the aforementioned issues. Standard SPH is basically designed to formulate the Navier–Stokes equations (Liu and Liu, 2003, 2010). So far only few studies have attempted to extend SPH to the shallow water equations. Wang and Shen (1999) investigated inviscid dam-break flows using SPH. Ata and Soulaimani (2005) proposed the stabilization term of SPH formulation. Rodriguez-Paz and Bonet (2005) presented a corrected variational SPH formulation for shallow water flows to conserve both the total mass and momentum. De Leffe et al. (2010) adopted an anisotropic kernel with variable smoothing length and performed SPH modeling of shallow-water coastal flows. Vacondio et al. (2011) used the characteristic boundary method into SPH formulation to simulate rectangular prismatic channel flows with open boundaries. Chang et al. (2011), and Kao and Chang (2012) applied SPH modeling to investigate shallow-water dambreak flows in realistic open channels and floodplains. Vacondio et al. (2012a,b) improved SPH for the closed boundary conditions by using virtual particles and introduced a particle-splitting procedure for addressing the issue of adequate particle-resolution in small-depth problems. Chang and Chang (2013) developed a new SPH scheme to solve the characteristic equations and to establish the open boundaries in non-rectangular and non-prismatic channel flows with the method of specified time intervals. Nevertheless, there still lacks research efforts that have utilized SPH to simulate the Boussinesq equations for investigating weakly nonhydrostatic shallow water flows.

To fill this gap, this study aims to develop a new SPH approach for investigating weakly nonhydrostatic shallow water flows. Firstly, the Lagrangian form of the 1D Boussinesq equations is derived. The numerical procedure of how to solve the above equations with SPH is given. Next, a comparison of the numerical results with the analytical solutions or published laboratory data is examined through four benchmark problems (solitary wave propagation, nonlinear interaction of two solitary waves, dambreak flow propagation, and undular bore development). The benefits and limits of the present SPH modeling are discussed.

2. Numerical method

Eqs. (4) and (5) are the 1D time-dependent hyperbolic system of partial differential equations. Generalized analytical solutions are not feasible for these equations. As a result, a numerical approach should be adopted to obtain the numerical solutions. In this study, a Lagrangian meshless SPH is adopted to solve Eqs. (4) and (5). In this numerical procedure, the pure space derivative terms (the hydrostatic and source terms) and the mixed space and time derivatives term (the material term of B_1 and B_2) of Eq. (5) are decomposed. The pure space derivative terms are explicitly solved through the predictor–corrector computational framework to give an intermediate value, and then the mixed space and time derivatives term is applied for computational correction with an implicit time-integration scheme.