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Derivation of rating curve by the Tsallis entropy

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SUMMARY

The stage–discharge relation, often called rating curve, is employed to determine discharge in natural and engineered channels. There are several methods for deriving a rating curve most of which are empirical. It is well recognized that rating curves are subjected to significant uncertainty, yet most of these methods do not have any provision to account for or do not quantify the uncertainty. This study employs the Tsallis entropy for deriving the rating curve, based on two simple constraints: (1) total probability and (2) mean discharge. Parameters of the derived curve are determined with the use of these two constraints. The rating curve is also determined by reparameterization with the use of an entropy parameter. The Tsallis entropy permits a probabilistic characterization of the rating curve and hence the probability density function of discharge underlying the curve. It also permits a quantitative assessment of the uncertainty of discharge obtained from the rating curve. The derived rating curve is found to be in agreement with field data and is also applied to ungaged watersheds. The rating curve is also extended beyond the range of discharge values used in its construction and its validity is then evaluated.

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1. Introduction

Rating curves are used for myriad purposes, including the determination of discharge for a prescribed stage, calibration of physically based hydraulic and hydrologic models, evaluation of flood inundation, and damage assessment (Singh, 1993). These curves are also used for constructing continuous records of discharge, continuous time series of sediment discharge or sediment concentration, continuous pollutant graphs, floodplain mapping, storage variation, hydraulic design, catchment routing, and damage assessment.

There are different types of rating curves, such as between stage–discharge relation, sediment rating curve (Kazama et al., 2005), pollutant rating curve, and drainage basin rating curve. Since rating curves are of similar form from an algebraic view point, fundamental to most rating curves is the estimation of discharge. Therefore, this study focuses on the stage–discharge rating curve only.

There are a multitude of methods for constructing rating curves, including graphical, hydraulic, artificial intelligence, and statistical. The graphical method is one of the commonly methods to construct a rating curve for a gaging site. It involves plotting observed discharge and stage data on a graph paper and fitting an equation to the data collected. Three types of rating curves, either of parabolic or power form, have been employed in practice. Parameters of these curves are determined either graphically or using a mathematical or statistical method, such as least-square method, maximum likelihood, pseudo-maximum likelihood, or segmentation (Petersen-Overleir and Reiten, 2005).

The hydraulic method involves the use of dimensional analysis or the use of the governing equations of mass and momentum conservation. Using dimensional analysis and the concept of self-similarity Baiamonte and Ferro (2007) derived a stage–discharge relation for flume measurements on a sloping channel. Liao and Knight (2007) suggested three formulae for rating curves for prismatic channels. Petersen-Overleir (2004) used nonlinear regression and Jones formula to account for hysteresis due to unsteady flow.

A simplified hydraulic approach is used by the U.S. Geological Survey (USGS) for estimating peak discharge in the absence of direct measurements, such as during floods. Discharge is determined from a 1-D flow model based on Manning's roughness and measurements of channel geometry, water surface elevations (Rantz, 1982). A similar method involves step-back water models and Manning's n for defining the shape of rating curves for stages where no measurements are made (Bailey and Ray, 1966). Indirect

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methods of discharge estimates entail extrapolates on estimated empirical roughness coefficients and are hence prone to error. The roughness coefficient can significantly vary (Jarrett, 1984).

Although a variety of hydraulic models exist, they rely on empirical roughness parameterization for a specific flow condition and do not express roughness as a function of stage. They may therefore not be able to accurately generate complete rating curve. Kean and Smith (2005) developed a hydraulic method for generating curves for geomorphologically stable channels. The method determines channel roughness from field measurements of channel geometry; the physical roughness of the bed, banks and floodplains; and vegetation density on the banks and floodplain. They applied the method to determine discharge at two USGS gaging stations on White Water River, Kansas, USA, which provided accurate discharge estimates.

In recent years artificial intelligence techniques have been employed for constructing rating curves. These include artificial neural networks (ANNs), genetic algorithm (GA), gene expression (GE), gene expression programming (GEP), and fuzzy logic. Bhattacharya and Solomatine (2000) used an artificial neural network for deriving a rating curve. Jain and Chalisgaonkar (2000) employed a three layered forward ANN, whereas Sudheer and Jain (2003) used an ANN with radial basis functions. Sahoo and Ray (2006) applied feed forward and back propagation and radial basis function ANNs for a stream in Hawaii. Guven and Aytek (2009) used genetic algorithm for Schuylkill River at Berne, Pennsylvania. Deka and Chandramouli (2003) compared an ANN, a modularized ANN, a conventional rating curve method and a neuro-fuzzy method for deriving rating curves. Bhattacharya and Solomatine (2005) found ANNs and M5 model tree to be more accurate for constructing rating curves. Habib and Meselhe (2006) used ANNs and regression analysis to derive rating curves.

Lohani et al. (2006) employed the Takagi-Sugano (T5) fuzzy inference system for deriving rating curves for Narmada River in India. Ghimire and Reddy (2010) compared GA and model tree 5 (M5) with gene expression programming (GEP), multiple linear regression and conventional stage–discharge relationship method. Azamathulla et al. (2011) compared GEP with GP, ANN and two conventional methods. Siavapragasam and Mutill (2005) employed a support vector machine (SVM) for extrapolating rating curves and applied it to three gaging stations in Washington and found SVM to be better than the widely used logarithmic method, a higher order polynomial and ANN.

Using the Shannon entropy (Shannon, 1948) Singh (2010d) derived the stage–discharge relation based on two simple constraints: (1) the total probability and (2) the mean logarithmic discharge. Parameters of the derived curves were determined with the use of these two constraints. The derived rating curves were tested using field data and were found to be in agreement with the curves obtained by the least square method. The entropy theory permitted a probabilistic characterization of the rating curve and permitted a quantitative assessment of the uncertainty of the rating curve.

However, a rating curve is often subject to uncertainties due to a number of factors: (1) errors in discharge measurements (Sauer and Meyer, 1992); (2) selection of a stable river cross-section; (3) maintenance of the stable cross-section; (4) abrupt changes in controls and submergence of controls causing irregularities in the slope of the stage–discharge relations; (5) variation in discharge for a given stage, due to variations in slope, velocity, or channel conditions, is small during the period of time involved; (6) lack of permanent control; and (7) existence of more than one control for high and low flows (Yoo and Park, 2010). The rating curve often changes with time and may not account for hysteresis in flow and therefore kinematic rating curves are not capable of representing looped conditions. Hence, it may not be accurate in determining streamflow when the stream bed profile and side slope characteristics change. Using measurements containing errors and outliers, Sefe (1996) derived a single routing curve for Ukavaiigo River at Mehembo, Botswana.

Herschy (1995) investigated errors in discharge due to errors in velocity and depth measurements. There can be a change in control from low flow to high flow, a segmentation method has been used to represent a rating curve (Overlier, 2006), suggesting an element of randomness in the stage–discharge curve. Hence, it will therefore be reasonable to argue that temporally averaged discharge can be treated as a random variable. Although significant temporal variability in discharge has been recognized, little effort has been made to account for its probabilistic characteristics when establishing rating curves and to quantify uncertainty in a rating curve. One way to accomplish the twin objectives of defining the probability distribution and the uncertainty of a rating curve is to use the entropy theory. This theory has an advantage over other methods in several respects. First, it takes account of the information available on the rating curve, such as moments (mean, variance, etc.) of discharge. These moments are more stable in time than individual measurements. Second, it permits one to quantify the information or uncertainty associated with the curve. Third, it paves the way to determine data sampling or the number of measurements needed to determine a robust rating curve. Fourth, it obviates the need for estimating the rating curve parameters empirically or by curve fitting. Fifth, since the parameters estimated by the entropy theory are expressed in terms of the specified constraints, they have physical meaning or they can be interpreted in terms of the given information. These considerations motivated the use of the Tsallis entropy theory, which is a generalization of the Shannon entropy theory.

The objective of this study therefore is to (1) derive, using the Tsallis entropy, the stage–discharge rating curve, (2) determine the rating curve parameters from the specified information expressed as constraints, (3) derive the probability density function associated with the three rating curves, (4) determine the entropy associated with these curves, (5) test the rating curve with field measurements, (6) apply the rating curve to ungauged watersheds, and (7) extend the rating curve beyond the range of discharge values used in its construction.

2. Forms of rating curves

A rating curve for a gage in a channel dominated by friction is normally expressed in a power form (Kennedy, 1964) as

$$Q = a(y - y_0)^b + c \tag{1}$$

where Q is the discharge (L^3/T, e.g., ft^3/s or m^3/s); y is the stage or height of water surface (L, e.g., ft or m); y_0 is the height (L) when discharge is negligible and is usually taken as a constant value or is sometimes used as a fitting parameter; b is exponent; a (L^{3-b}/T) and c (L^3/T) are parameters; here L is the length dimension and T is the time dimension. Eq. (1) is a general form and specializes into three popular types that are commonly employed (Corbett, 1962). Although the three forms differ from each other through parameters, these forms have been popularly used and reported separately in the hydraulic literature (Singh, 1996) and stem from river morphological characteristics. Therefore, they are described as such.

Type 1: In this case, $y_0 = 0$ and $c = 0$. Eq. (1) then becomes

$$Q = ay^b \tag{2a}$$

or in logarithmic form

$$\log Q = \log a + b \log y \tag{2b}$$