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Impact of diffusion coefficient averaging on solution accuracy of the 2D nonlinear diffusive wave equation for floodplain inundation

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SUMMARY

In the study, the averaging technique of diffusion coefficients in the two-dimensional nonlinear diffusive wave equation applied to the floodplain inundation is presented. As a method of solution, the splitting technique and the modified finite element method with linear shape functions are used. On the stage of spatial integration, it is often assumed that diffusion coefficient is constant over element and equal to its average value. However, the numerical experiments indicate that in the case of the flow over the dry floodplain with sudden changes in depths an inadequate averaging of these coefficients can lead to a non-physical solution or even to its instability. In the paper, the averaging techniques for estimation of diffusion coefficients were examined using the arithmetic, geometric, harmonic and the direction dependent means. The numerical tests were carried out for the flows over initially dry floodplain with varied elevation of bottom. It was shown that the averaging method based on the arithmetic mean with respect to the diffusion coefficients provides the satisfactory results in comparison to other techniques.

1. Introduction

Numerical simulation of floodplain inundation process is one of the most important problems in hydrological practice. In order to predict the propagation of flood wave over an initially dry area the shallow water equations (SWE) are frequently used (Heniche et al., 2000; Horritt, 2002; Horritt and Bates, 2001; Liang and Borthwick, 2009). In many cases, information on the extent of inundation can be also acquired using a simplified model in the form of diffusive wave equation (Hsu et al., 2000; Moussa and Bocquillon, 2009; Szymkiewicz and Gąsiorowski, 2012). This equation obtained by neglecting of the inertial force in the SWE was proposed by Hromadka and Yen (1986). For the two-dimensional (2D) case the diffusive wave equation takes the following form:

$$\frac{\partial H}{\partial t} - \frac{\partial}{\partial x}\left(K_x \frac{\partial H}{\partial x}\right) - \frac{\partial}{\partial y}\left(K_y \frac{\partial H}{\partial y}\right) = 0 \tag{1}$$

where x, y are space co-ordinates, t is the time, H is the water surface elevation above the assumed datum, and K_x, K_y are the coefficients of diffusion in x and y direction respectively.

The coefficients K_x and K_y are defined as:

$$K_x = \frac{1}{n} h^{5/3} \left|\frac{\partial H}{\partial x}\right|^{-1/2}, \quad K_y = \frac{1}{n} h^{5/3} \left|\frac{\partial H}{\partial y}\right|^{-1/2} \tag{2a, b}$$

where $h = H–Z$ is the flow depth, Z is the bottom elevation above the assumed datum, and n is the Manning roughness coefficient.

The 2D nonlinear diffusive wave Eq. (1) is classified as a partial differential equation of 2nd order of parabolic type, where both coefficients of diffusion depend in a nonlinear manner on the water depth h, bottom elevation Z, water level H as well as the derivatives $\partial H/\partial x$ and $\partial H/\partial y$. The nonlinear character of diffusive wave equation causes complications in its numerical solution, in particular for overland flow problems. For this reason Eq. (1) requires a choice of adequate numerical methods.

A couple of numerical approaches have been proposed to solve the 2D diffusive wave equation. For example, Hromadka and Yen (1986), Han et al. (1998) and Lal (1998) used the finite difference method (FDM) in the form of nodal domain integration method, whereas Zhang et al. (2004) applied the FDM method with irregular triangle mesh. Di Giammarco et al. (1996) presented an alternative formulation for the mixed 1–2D overland flow using the conservative volume finite element method (CVFE), and Prestininzi, 2008 developed the finite volume method (FVM) in combination with the storage cell scheme for dam-break induced

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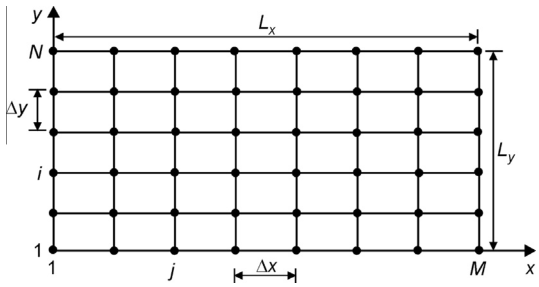

Fig. 1. Rectangular mesh covering the solution domain.

flow. The diffusive wave equation has been also solved using the modified Galerkin finite element method (FEM). This approach has been successfully applied to the solution of 2D problem using triangular elements (Szymkiewicz and Gąsiorowski, 2012) as well as to the solution of the one-dimensional diffusive equations (Gąsiorowski, 2013). A modification of the spatial integration procedure in FEM leads to a more general algorithm including a weighting parameter. Due to this modification the resulting algorithm allows to control the accuracy and stability of solution, thereby it is possible to achieve a solution without numerical oscillations.

An effective solution of the 2D diffusive wave equation has been obtained using the time splitting technique such as the Alternating Direction Explicit (ADE) method or the Alternating Direction Implicit (ADI) method (Lal, 1998). Apart from the time splitting algorithms, we can also distinguish the approach based on the dimensional splitting (Szymkiewicz, 1993; LeVeque, 2002). According to this method the splitting process is performed with respect to the space independent variables. Consequently, in each time step the 2D diffusive wave equation is solved as two 1D sub-problems for each direction separately (Gąsiorowski, 2013). This procedure leads to a more effective algorithm of solution with tri-diagonal systems of algebraic equations. Moreover, in order to simplify the discretization issues the solution domain is represented by a rectangle containing the actual domain of interest, which may be of irregular shape. The only restriction resulting from the procedure is that considered domain should be large enough to include the whole area to be wetted in the simulated event.

All the above numerical methods (FEM, FDM and FVM) provide an approximate solution of the diffusive wave equation in the nodes of the mesh only. Regardless of the applied method, each of them requires an adequate estimation of the diffusion coefficients referred to the area between adjacent nodes. The choice of averaging method must be considered thoroughly, especially when the overland flow problem is simulated. One of the most popular approach for averaging of the diffusion coefficients is the arithmetic mean with respect to the water depth (Singh, 1996; Lal, 1998; Szymkiewicz and Gąsiorowski, 2012). It gives correct results for typical overland flow simulation problems. However, numerical experiments show that this kind of averaging provides the unexpected effects in solution when the flow occurs on the dry floodplain in the vicinity of obstacles. In such situations an inadequate approximation of the diffusion coefficients leads to an unstable or non-physical solution with water at rest even with non-zero water surface slope. For this reason it is necessary to find other alternative methods for averaging of the diffusion coefficients while analyzing the floodplain inundation problems.

As one can notice the nonlinear diffusive wave Eq. (1) is very similar to the Richards equation describing the flow in unsaturated zone of the porous media (Weill et al., 2009; Lal, 1998). In the case of the Richards equation the averaging of the water permeability was studied by several authors (Haverkamp and Vauclin, 1979; Zaidel and Russo, 1992; Miller and Williams, 1998; Belfort and Lehmann, 2005; Szymkiewicz, 2009; Szymkiewicz and Hemling, 2011). There are various averaging techniques for hydraulic conductivity and a comprehensive presentation focusing on the solution of 1D Richards equation is given by Szymkiewicz (2012). As it has been presented in the literature these averaging methods can differ significantly in their predicted results, especially when the solution contains a steep wetting front. For this reasons, it seems to be justified to take into account some suggestions resulting from modeling of the flow in unsaturated media and to implement them for the considered diffusive wave equation describing the overland flow.

The objective of this paper is the analysis of various averaging techniques of the diffusion coefficients appearing in 2D nonlinear diffusive wave equation. All further considerations are related to the case when the governing equation is solved numerically using the splitting technique and the modified FEM.

2. Splitting process for 2D diffusive wave equation

In order to ensure a more effective and flexible algorithm the 2D diffusive wave Eq. (1) can be split with regard to the space independent variables x and y (Gąsiorowski, 2013). According to this method in each time step the 2D problem is split and reduced to the solution of two 1D subproblems describing separately the propagation process in x and y direction respectively.

It is assumed that the considered domain has the form of rectangle of dimension $L_x \times L_y$ (Fig. 1) and it is covered by a mesh given by intersection of two families of straight lines.

The first one is parallel to the x axis and the second one is parallel to the y axis. The mesh constituted by M columns spaced with Δx and N rows spaced with Δy contains $N \times M$ nodes as it is shown in Fig. 1. According to the splitting method the solution of Eq. (1) runs in two stages for each time step Δt. At the first stage a set of 1D equations in x direction:

$$\frac{\partial H^{(1)}}{\partial t} - \frac{\partial}{\partial x}\left(K_x \frac{\partial H^{(1)}}{\partial x}\right) = 0 \tag{3}$$

corresponding to each row is integrated with the initial condition $H^{(1)}(x, y, t) = H(x, y, t)$. This equation is solved for all rows $i = 1, 2, \ldots, N$. The solution $H^{(1)}(x, y, t + \Delta t)$ of Eq. (3) in the current time step is used at the second stage as the initial condition $H^{(2)}(x, y, t) = H^{(1)}(x, y, t + \Delta t)$ for the solution of a set of 1D equations in y direction:

$$\frac{\partial H^{(2)}}{\partial t} - \frac{\partial}{\partial y}\left(K_y \frac{\partial H^{(2)}}{\partial y}\right) = 0 \tag{4}$$

corresponding to each column. The integration of Eq. (4) is performed for each column $j = 1, 2, \ldots, M$ and gives the final solution at the next time level $H(x, y, t + \Delta t) = H^{(2)}(x, y, t + \Delta t)$.

In the case of linear equations we can apply the superposition principle. Consequently, the order of solution of Eqs. (3) and (4) is not important for the solution accuracy. However, for the nonlinear systems this property is not valid. Thus, if the splitting method is applied to the non-linear equation then the splitting error can be observed in its numerical solution. It depends on the order of accuracy of the algorithms applied to compute subproblems and it is connected with spatial flow orientation. For example, in considered 2D diffusive equation when the flow is oriented approximately along the x axis, starting computations with x and next with y direction we can obtain a greater splitting error than in the opposite case, i.e. starting with y and next x direction. The splitting error has a numerical character and depends on the size