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Stochastic description of infiltration between aquifers

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SUMMARY

Aim of this work is to propose a stochastic description of the leakage between two aquifers separated by a semi-permeable layer with low hydraulic conductivity. The source of uncertainty here considered is the random fluctuation of the phreatic surface of surficial aquifer, originated from random rainfall events. The study focuses on an area surrounding a pumping well penetrating the deep aquifer and impacting its piezometric level, where infiltration from the surficial aquifer can be more harmful. Closed form expressions for the leakage between the surficial and the deep aquifer are used to obtain the long-term probability distribution of leakage flow rate, assuming the shallow phreatic surface dynamics modeled with a Poisson-driven stochastic process. A sensitivity analysis is performed to verify the variability of the probability distribution of leakage within the range of feasible parameter values, then the stochastic model is applied to three field cases where time series of the piezometric levels of the phreatic aquifer are available. Results show that the induced variability of the discharge flowing between aquifers is remarkable and that in general it cannot be neglected despite the low hydraulic conductivity of the semi-permeable layer. The proposed probabilistic model is a useful tool for evaluating the risk associated to contaminant transport into deep aquifers and its fate in relation to groundwater withdrawals.

1. Introduction

Groundwater resource is of great importance for human activities and life, from agricultural uses and industrial processes to drinking requirements. The most precious groundwater is that of deep aquifers, because usually it is less contaminated and, in fact, most of the water supply systems pump water from confined aquifers.

In nature, many deep aquifers are not perfectly separated from the overlying aquifer because the layer dividing them is not completely impermeable and/or because of the presence of abandoned wells and boreholes that may allow for some leakage. In these circumstances, leakage occurs from an aquifer to the other, upward or downward, according to the piezometric levels of the aquifers. When a confined aquifer receives some recharge infiltrating from the overlying phreatic aquifer, it is referred to as a semi-confined aquifer and quantification of such infiltration is important for management purposes, in order to evaluate both the effects on the hydraulic head and velocity fields of the underlying aquifer and, if a contaminant is released, the mass of solute leaching from the surface to the deep aquifer.

Leaving aside the infiltration caused by abandoned wells and improperly plugged boreholes (e.g., Avci, 1994; Lacombe et al., 1995; Nordbotten et al., 2004) and limiting the analysis to the leakage through the semi-permeable layer, the infiltration flow depends on the difference between piezometric levels of the unconfined and semi-confined aquifers and on the thickness and hydraulic conductivity of the aquitard (or semi-permeable layer). These quantities may be regarded as deterministic or as space–time random factors when taking into account their natural variability. For instance, the random nature of the hydrogeological parameters has been accounted for in the past to investigate the effects of leakage on the flow field of the semi-confined aquifer. Examples include: Li and Graham (1998, 1999) who dealt with the stochastic analysis of solute transport in heterogeneous semi-confined aquifers subject to spatiotemporal random recharge; Butera and Tanda (1999) who, following the linear theory of Dagan (1984), developed a stochastic approach to describe the effect of a uniform recharge on semi-confined aquifers with heterogeneous transmissivity, with the aim of characterizing the velocity field and the transport processes. Lu and Zhang (2002) investigated through a spectral method the non-stationarity features of heterogeneous semi-confined aquifers. More recently Yeh and Chang (2009) applied the nonstationary spectral approach to quantify the variability of hydraulic head and velocities in deep heterogeneous aquifers subject to spatial and periodic recharge, as influenced by the water table fluctuations in the overlying phreatic aquifer. In addition, attention has been paid to the development of methods for computing pumping-induced recharge: Butler and Tsou (2003) outlined an approach for calculating the volume of pumping-induced leakage for both infinite and bounded homogeneous aquifers, assuming a constant head in the upper aquifer. Zhan and Bian (2006) derived closed-form analytical solutions of the

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steady state and leakage volumes within a given circular area for both constant-rate and constant-drawdown pumping wells, in homogeneous aquifers.

To our knowledge, less attention has been paid to the characterization of leakage as driven by fluctuations of the water table in the phreatic aquifer. When these fluctuations are induced by random rainfall events, the shallow water table dynamics becomes itself stochastic and new (probabilistic) frameworks are necessary to describe the manifold interactions and feedbacks between groundwater, climate, soil and vegetation (e.g., Rodriguez-Iturbe et al., 2007).

Some probabilistic frameworks used in environmental sciences and ecohydrology are detailed by Ridolfi et al. (2011) considering different characteristics of the random forcing (Poisson noise, Gaussian white noise, dichotomous noise, etc.). An example of a probabilistic soil water balance driven by rainfall events is found in Rodriguez-Iturbe et al. (1999), while the soil water balance in the presence of an aquifer has been considered, for example, by Salvucci and Entekhabi (1994), Bierkens (1998), Laio et al. (2009), and Tamea et al. (2010). None of such contributions considers the effect of stochasticity on deep aquifers, nor the presence of subsurface geological structures, such as soil layers having different hydraulic conductivity. The goal of the present work is to propose a probabilistic framework describing the infiltration between aquifers, separated by a semi-permeable layer, as driven by random fluctuations of the shallow water table induced by rainfall events.

The proposed model assumes some simplifying hypothesis about the pedology and hydrogeology of the site and about the dynamics of the shallow water table. It then uses a simple and known groundwater solution to describe the flow leaching through the semi-permeable layer in an area surrounding a well, which withdraws water from the deep aquifer and impacts the vertical flow by modifying its piezometric surface. After reconstructing the probability distribution of the phreatic surface, the model returns a probability density function of the flow rate infiltrating over the considered area, under the assumption that infiltration velocities are low enough that the influence of leakage on the piezometric surfaces of the two aquifers is negligible.

The probabilistic description of leakage is of large interest for several reasons: (i) it helps understanding the relevance of shallow water table fluctuations in the presence of a pumping well withdrawing water from deep aquifers; (ii) it allows one to quantify the leakage flow variability as a function of rainfall variability, enabling for example a quantitative analysis of the effect of some climate changes; and finally, (iii) it gives a tool for evaluating the risk of pollution of deep aquifers or, in general, the fate of transport associated to leakage. Given the relevance of these aspects, the present paper aims at proposing a useful tool for probabilistic assessments in subsurface hydrology.

The paper is so organized: the problem is initially presented and the infiltration flow rate quantified deterministically, next the probabilistic description of the piezometric fluctuations of the phreatic aquifer and of infiltration follows, finally three case studies are presented and discussed.

2. Problem description

Let us consider the case of two homogeneous, isotropic and indefinite aquifers separated by a horizontal semi-permeable soil layer which allows for some water infiltration between the two strata. The surficial aquifer (identified by subscript "1") is characterized by a horizontal phreatic surface lying at a positive depth from the soil surface, $h_1(t)$; such depth may change in time due to the fluctuations induced by rainfall events and it has a maximum value indicated by $h_{1,0}$. The deep aquifer (identified by subscript "2") has a piezometric surface which, in the absence of perturbations, lies at a fixed (positive) depth, $h_{2,0}$, from the soil surface. In general such depth is different from the surficial phreatic surface thus generating a flow through the semi-permeable layer, from one aquifer to the other.

A fully penetrating well of radius r_0 withdraws water from the semi-confined aquifer, lowering its piezometric surface at a depth $h_2(r)$ which varies with the distance, r, from the center of the well (Fig. 1). The drawdown in the piezometric head due to the well, $\Delta_w(r)$, never lowers the piezometric surface below the top of the deep aquifer and it becomes negligible at a distance R identifying the radius of influence of the well, while the rate of water pumped out from the well, Q_p, is assumed to be constant in time.

The semi-permeable layer separating the two aquifers allows the water to flow either downward or upward, depending on the relative position of the two piezometric surfaces. According to the Darcy's law, the overall flow rate $Q(t)$ through the semi-permeable layer is a function of the characteristics of the layer and of the piezometric head difference in the two aquifers. Focusing on the vertical flow within the radius of influence of the well and assuming the domain to be axial-symmetric without other wells or disturbances nearby, the discharge can be quantified by

$$Q(t) = \int_{r_0}^{R} \frac{K_L}{b_L} \cdot (h_2(r) - h_1(t)) \cdot 2\pi r \cdot dr, \qquad (1)$$

where K_L is the hydraulic conductivity of the semi-permeable layer and b_L its thickness, both assumed to be uniform in space. If the piezometric surface of the deep aquifer has a constant gradient, the following analysis is still valid, provided that $h_{2,0}$ is taken as the average piezometric depth in the area of radius R.

It should be noted that the hydraulic conductivity of the semi-permeable layer is here taken small enough to prevent the phreatic surface (of surficial aquifer) from being influenced by the well and to avoid that leakage significantly modifies the pressure regime of the deep aquifer. The decoupling of the dynamical systems of shallow and deep aquifers is necessary for the application of the leakage model (end of Section 2.1) and the development of the stochastic framework presented in Section 3.1.

2.1. Components of the infiltrating flow

Three contributions to the infiltrating flow can be identified by separating three different components of the piezometric head difference in (1).

Modelling the fluctuations of the phreatic surface as stochastic upward displacements from the deepest position (see Section 3), the first component of the piezometric head difference in (1) is given by the vertical distance between the (time-varying) position of the phreatic surface and its maximum depth, i.e., $\Delta_f(t) = h_{1,0} - h_1(t)$. Such difference is, by construction, always positive and the phreatic surface is here assumed to increase simultaneously in the area of influence of the well, thus the contribution to infiltration is uniform in space.

The second component of the piezometric head expresses the head difference between the two aquifers, in the absence of phreatic-surface fluctuations and of a drawdown imposed by the well. This component is given by the vertical distance between the maximum depth of the phreatic surface and the undisturbed position of the piezometric surface in the deep aquifer, i.e., $\Delta_0 = h_{2,0} - h_{1,0}$. Such difference is constant in time and uniform in space, and it can take positive or negative values according to the relative position of the piezometric head in the two aquifers. In case of a constant piezometric gradient in the deep aquifer, such difference would not be uniform, but using the spatial average of $h_{2,0}$ the