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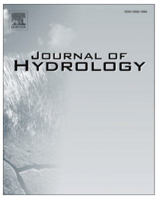

Estimating recharge rate from groundwater age using a simplified analytical approach: Applicability and error estimation in heterogeneous porous media

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SUMMARY

Environmental tracer data is commonly coupled with a simplified analytical model (e.g., exponential model) to interpret the aquifer recharge rate from tracer-based groundwater age. Can we still use this approach if the aquifer is heterogeneous? In this study we use a series of demonstrative numerical simulations to better understand how heterogeneity influences the spatial distribution of groundwater age and the interpretation of the recharge rate within an unconfined aquifer. Eight discontinuous horizontal lenses of contrasting hydraulic conductivity were arbitrarily added to a homogeneous base case. The apparent recharge rate was calculated at each node using the simulated mean age in the exponential analytical solution. The apparent recharge rate in the heterogeneous cases was then compared to the known simulated recharge rate to quantify the magnitude of error and its distribution in the flow field. For demonstration purposes, the reasonable application of the exponential model in heterogeneous aquifers is constrained to the case where the absolute error in the estimated recharge rate is $\leqslant 25\%$. From the simulations conducted, lenses with a contrasting hydraulic conductivity of ten times or less had little impact on obtaining useable recharge rate estimates (absolute error $\leqslant 25\%$ in $\leqslant 10\%$ of the domain). Conversely, hydraulic conductivity contrasts greater than ten times had a significant impact on perturbing the flow field and inducing geometrically complex and disconnected areas of under and overestimation in the interpreted recharge rate (absolute error $\leqslant 25\%$ reduced to as low as 39% of the domain). The reduced suitability of the exponential model in this case can be partly overcome by convergence in the average recharge rate obtained from unbiased samples collected from multiple locations.

1. Introduction

Environmental tracers (e.g., ^{14}C, ^{3}H/^{3}He, ^{36}Cl, chlorofluorocarbons) are commonly used to estimate the age of groundwater and constrain recharge rates of aquifers. Recharge is deduced from age using simplified analytical solutions for different aquifer geometries (e.g., piston flow model, exponential model) that assume uniform hydraulic conductivity and porosity. Cook and Böhlke (1999) present these lumped-parameter models in detail. This study will focus on the use of the commonly used exponential model [e.g., Vogel (1967) solution] – an approximation that describes the depth-dependent vertical age stratification within an unconfined aquifer of known dimensions and porosity for a given recharge rate.

The exponential model continues to be employed in a variety of geologic/hydrogeologic settings, including: lacustrine and fluvial deposits with interconnected sand, gravel, and silt lenses (Harrington et al., 2002); unconsolidated (glacial and nonglacial sand and gravel), semiconsolidated, and fractured bedrock aquifers across the United States (results compiled by McMahon et al., 2011); complex volcanic rock with interflow structures overlying a hydro-volcanic sedimentary formation (Hagedorn et al., 2011); and a shallow homogeneous sandy alluvium (Hinkle et al., 2007). It is clear that the exponential model is being applied in practice but in settings where the geologic conditions are not likely to meet the fundamental assumptions underpinning the formulation of the analytical model, namely with respect to homogeneous hydraulic conductivity. What is the effect of the mismatch between implicit or explicit assumptions in the analytical approach and the conditions under which it is applied, and under what hydrogeologic conditions does it matter?

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Previous studies have identified uncertainty in tracer-based groundwater ages and interpreted velocities because of aquifer heterogeneity and groundwater mixing due to hydrodynamic dispersion (Varni and Carrera, 1998; Weissmann et al., 2002). Both deterministic and stochastic approaches have been used to generate heterogeneous hydraulic conductivity fields in numerical investigations (Varni and Carrera, 1998; Troldborg et al., 2007; Larocque et al., 2009; McCallum et al., 2013). What is lacking is a systematic evaluation of how heterogeneity influences the utility of simplified interpretive solutions, such as the exponential model, in the estimation of recharge rates from environmental tracers, including the quantification of error.

This demonstrative study examines age distributions and recharge rate error when applying the exponential solution in a heterogeneous unconfined aquifer using numerical models. The objectives are to: (1) examine the influence of the heterogeneity on spatial distribution of groundwater age, and (2) compare interpreted with "known" recharge rates to quantify the error distribution within a simple and systematic framework. The simulations were designed to represent uniform recharge into a medium-grained sand aquifer sparsely interbedded with eight thin and arbitrarily distributed lenses ranging in grain size from silt to coarse sand. The intention is not to provide generic fixed error bars or correction factors that can be applied to exponential model-derived recharge rate estimates should aquifer heterogeneity be noted at the field site. Rather, the intention is help build some intuition on what aquifer heterogeneity can do to groundwater age distributions, what the implications are for field-based research, and better identify the hydrogeologic conditions under which simplified analytical solutions might be reasonably applied in heterogeneous aquifers using demonstrative rather than exhaustive simulations.

2. Methodology

2.1. Theory

Fig. 1A provides a schematic of the exponential model domain [e.g., Vogel (1967)] including the geometry and boundary conditions. The time since recharge t at a point in the aquifer is related to the average recharge rate between the point of entry and the sampling point $\overline{R}$ in a homogeneous, isotropic aquifer by:

$$\overline{R} = \frac{H\theta}{t}\ln\left(\frac{H}{h}\right) \tag{1}$$

where H is the aquifer thickness, θ is the aquifer porosity, and h is the vertical distance from the base of the aquifer (equal to H minus depth z). Groundwater age is vertically stratified, increases logarithmically with depth, and is independent of the distance to the groundwater flow divide. If the average recharge rate between the point of entry and the groundwater flow divide $\overline{R}'$ is equal to $\overline{R}$ (Fig. 1a), which is the case in this study, then the ratio of the distances d:X is the same as the ratio of the depths z:H, giving (Harrington et al., 2002):

$$d = \frac{zX}{H} \tag{2}$$

2.2. Numerical implementation

Fig. 1B provides a schematic of the conceptual model and two-dimensional (2-D) simulation domain, including dimensions and boundary conditions. Numerical simulations were conducted using HydroGeoSphere (HGS) (Therrien et al., 2006). The solution domain is similar to one employed by Solomon and Sudicky (1991) to investigate the influence of hydrodynamic dispersion on the distribution of apparent groundwater age using tritium and helium 3 isotope ratios (^{3}H/^{3}He). The solution domain was discretized into 0.25 m × 0.25 m blocks of unit thickness (324,162 nodes; 160,000 elements) based on grid convergence tests and obtaining target grid Peclet and Courant numbers less than two. Eight discrete lenses (0.5 m × 50 m, 2 elements × 200 elements) were arbitrarily distributed throughout the domain (see locations in Fig. 1B). A uniform recharge R of 0.1 m/y was applied to the top surface of the domain.

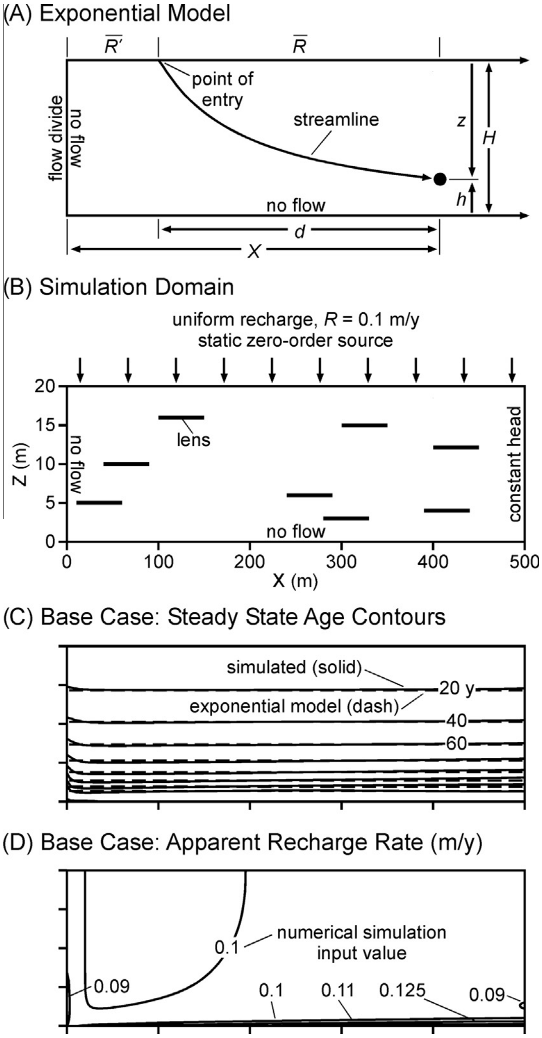

Fig. 1. (A) Schematic of the exponential model domain. (B) Conceptualization of the simulated aquifer including dimensions and boundary conditions. (C) Simulated (dashed) and exponential model (solid) steady-state groundwater age contours (20 y contour intervals) for the homogeneous base case ($K_{l/b} = 1$). (D) The apparent recharge rate in the base case solution domain derived using Eq. (4). Flow is left to right. The vertical exaggeration is 8.5.

The simulations were designed to represent a simple background medium-grained sand aquifer (hydraulic conductivity $K_{background} = 1 \times 10^{-5}$ m/s) with interbedded lenses ranging from silt to coarse sand (hydraulic conductivity $K_{lens} = 1 \times 10^{-8}$ m/s to 1×10^{-2} m/s), all with a porosity θ of 0.3. The hydraulic conductivity of the lenses was the only property to change between the scenarios, and all lenses had the same K_{lens} for a given case. The ratios of K_{lens} to $K_{background}$ (denoted $K_{l/b}$) tested were 0.001, 0.01, 0.1, 1 (homogeneous base case, no lenses), 10, 100, and 1000.

All transport simulations were performed in a steady-state flow field. The groundwater age distribution was simulated using the mean age approach (Goode, 1996). Particles of water enter the