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Three-dimensional steady-state Green's functions for fluid-saturated, transversely isotropic, poroelastic bimaterials

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SUMMARY

Green's function for poroelastic bimaterials is the foundation to study the interaction of fluid and solid in poroelastic materials. For this object, we first summarize the compact general solutions of fluid-saturated, transversely isotropic, poroelastic materials in terms of harmonic functions. Based on these compact general solutions, the three-dimensional Green's function for fluid-saturated, transversely isotropic, poroelastic bimaterials under a steady-state point fluid source is solved by introducing six new harmonic functions. All poroelastic components are expressed in terms of elementary functions and are convenient to use. Numerical results are given graphically by contours and some conclusions for the pore fluid pressure and stress distributions are obtained.

1. Introduction

The study of the deformation in porous saturated materials (Biot, 1955, 1956) is a subject of interest in the field of hydrology. As a primary problem, the Green's functions of poroelastic material play an important role in both applied and theoretical studies on the hydrology, especially the studies of interactions between the water of the rivers, lakes and seas with the saturated, poroelastic water-beds. Firstly, the Green's functions are essential for the boundary element method, which is an efficient numerical tool to study the above hydrologic problems. Secondly, by the method of superposition, the Green's functions can be used to search for the analytical solutions of the above hydrologic problems. By this method, one can find the analytical solutions for sources distributed over an arbitrary region by integrating the Green's functions over this region.

In addition, the saturated, poroelastic water-beds are always constituted of different poroelastic materials. This constitution will lead to the obvious interface effects, which are often the main reasons resulting in the move and failure of the water-beds. In this case, as the primary foundation to study the interface effects, The Green's functions for saturated, poroelastic bimaterials can largely benefit the analyses of this kind of hydrologic problems.

For isotropic materials, Banerjee and Butterfield (1982) presented the well-known closed-form Kelvin fundamental solution. For transversely isotropic materials, Lifshitz and Rozentsveig (1947) and Lejcek (1969) derived the Green's functions using the Fourier transform method. Elliott (1948), Kroner (1953) and Wills (1965) obtained them using the direct method and Sveklo (1969) found them using the complex method. Pan and Chou (1976) solved the Green's function in the form of compact elementary functions. For anisotropic materials, Pan and Yuan (2000) and Pan (2003) obtained the three-dimensional Green's functions for bimaterials with perfect and imperfect interfaces, respectively.

For poroelastic material, Cleary (1977) derived the Green's functions for an infinite fluid-saturated, isotropic, poroelastic material. Rudnicki (1980) corrected some errors in those solutions for a point force and fluid mass source. Rajapakse and Senjuntichai (1993) obtained the Green's functions for a semi-infinite fluid-saturated, isotropic, poroelastic material. Watanabe and Kurashige (1997) derived the solutions for an isotropic, poroelastic material with vanishing permeability in one direction by using the Laplace and Fourier transform method. Using the same approach, Ganbe and Kurashige (2000) obtained the Green's functions for an isotropic, poroelastic material of transversely isotropic permeability. Using Kupradze's method (1979), Kazi-Aoual et al. (1988) sought for the Green's functions for an infinite fluid-saturated,

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transversely isotropic, poroelastic material. However, the solutions in an explicit form are not presented. Employing the Kupradze's method together with the triple Fourier transforms and Hankel transforms, Taguchi and Kurashige (2002) presented the Green's functions for an infinite fluid-saturated, transversely isotropic, poroelastic material.

There are two important working states for poroelastic materials are studied in above literatures. One is the steady-state in which the poroelastic loading varies slowly with time and the rate of fluid mass content vanishes. Another one is the transient-state in which the poroelastic loading varies quickly with time and the rate of fluid mass content does not vanish. Both states exist in poroelastic water-beds and are all necessary to be studied. The steady-state often exists in the areas which are always under the water, while the transient-state often exists in the areas which are near the sides of water-beds.

In this paper, the three-dimensional steady-state Green's function for a point fluid source in a fluid-saturated, transversely isotropic, poroelastic bimaterials is investigated. For completeness, the general solution is summarized in Section 2 based on the works of Chen et al. (2004) and Li et al. (2010). In Section 3, six new harmonic functions are constructed in terms of elementary functions with undetermined constants. The corresponding poroelastic field can be obtained by substituting these functions into the general solutions after determining the constants by the compatibility and equilibrium conditions. Numerical examples are presented in Section 4. The contours of the pore fluid pressure and stress components are shown graphically. Finally, the paper is concluded in Section 5.

2. General solution

Consider a fluid-saturated, fully transversely isotropic, poroelastic material in Cartesian coordinates (x,y,z). When the plane of isotropy is parallel to the plane xoy, the constitutive relations are (Cheng, 1997)

$$\sigma_x = c_{11}\frac{\partial u_x}{\partial x} + c_{12}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} - \alpha_1 p, \quad \tau_{yz} = c_{44}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right),$$
$$\sigma_y = c_{12}\frac{\partial u_x}{\partial x} + c_{11}\frac{\partial u_y}{\partial y} + c_{13}\frac{\partial u_z}{\partial z} - \alpha_1 p, \quad \tau_{zx} = c_{44}\left(\frac{\partial u_x}{\partial z} + \frac{\partial u_z}{\partial x}\right),$$
$$\sigma_z = c_{13}\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) + c_{33}\frac{\partial u_z}{\partial z} - \alpha_3 p, \quad \tau_{xy} = c_{66}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right), \tag{1a}$$

$$p = \lambda\zeta - \alpha_1(\sigma_x + \sigma_y) - \alpha_3\sigma_z, \tag{1b}$$

where $u_i\,(i = x,y,z)$ are the components of mechanical displacement; σ_i and $\tau_{ij}\,(i,j = x,y,z)$ are the components of normal stress and shear stress, respectively; p and ζ are the pore fluid pressure and the rate of pore fluid mass content, respectively; λ and $\alpha_i\,(i = 1,3)$ are the Biot coefficient and Biot effective stress coefficient, respectively; $c_{ij}\,(i,j = 1,2,3,4,6)$ are the elastic moduli. $c_{66} = (c_{11} - c_{12})/2$ is held for transversely isotropic, poroelastic materials.

It should be noted that elastic moduli c_{ij} and Biot coefficient α_i are in relationships with engineering elastic moduli as follows:

$$c_{11} = \frac{E(E' - E\nu'^2)}{(1+\nu)(E' - E'\nu - 2E\nu'^2)}, \quad c_{12} = \frac{E(E'\nu + E\nu'^2)}{(1+\nu)(E' - E'\nu - 2E\nu'^2)},$$
$$c_{13} = \frac{EE'\nu}{E' - E'\nu - 2E\nu'^2}, \quad c_{33} = \frac{E'^2(1-\nu)}{E' - E'\nu - 2E\nu'^2},$$
$$c_{44} = G' = \frac{E'}{2(1+\nu')}, \quad c_{66} = G = \frac{E}{2(1+\nu)}, \tag{2a}$$

$$\alpha_1 = 1 - \frac{c_{11} + c_{12} + c_{13}}{3K}, \quad \alpha_3 = 1 - \frac{2c_{13} + c_{33}}{3K}, \tag{2b}$$

where E, G and E', G' are the in-plane and out-of-plane elastic moduli, respectively; ν is the drained Poisson's ratio characterizing the transverse strain reduction in the plane of isotropy due to a tensile stress in the same plane, and ν' is the drained Poisson's ratio corresponding to the transverse strain reduction in the plane of isotropy due to a tensile stress normal to it; K is the bulk modulus of solid skeleton.

In the absence of body forces, the mechanical equilibrium equations are

$$\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{zx}}{\partial z} = 0, \quad \frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} = 0, \quad \frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0. \tag{3a}$$

In the following analysis, uncoupled poroelastic theory is adopted by assuming that the poroelastic loading varies slowly with time and the rate of fluid mass content vanishes. Consequently, the poroelastic material is in a steady-state, the pore fluid pressure field is constant in time and governed by the following Laplace equations.

$$\left[\kappa_1\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) + \kappa_3\frac{\partial^2}{\partial z^2}\right]p = 0, \tag{3b}$$

where $\kappa_i \quad (i = 1,3)$ are permeability and $\kappa_2 = \kappa_1$ when the body is isotropy in the xy-plane.

Parallel to Chen et al. (2004), the general solutions to Eqs. (1) and (3) can be induced as follows:

$$U = \Lambda\left(\mathrm{i}\psi_0 + \sum_{j=1}^{3}\psi_j\right), \quad u_z = \sum_{j=1}^{3}s_j k_{1j}\frac{\partial\psi_j}{\partial z_j}, \quad p = k_{23}\frac{\partial^2\psi_3}{\partial z_3^2}, \tag{4a}$$

$$\sigma_1 = 2\sum_{j=1}^{3}\left(c_{66} - \omega_j s_j^2\right)\frac{\partial^2\psi_j}{\partial z_j^2} = -2\sum_{j=1}^{3}\left(c_{66} - \omega_j s_j^2\right)\Delta\psi_j,$$
$$\sigma_2 = 2c_{66}\Lambda^2\left(\mathrm{i}\psi_0 + \sum_{j=1}^{3}\psi_j\right),$$
$$\sigma_z = \sum_{j=1}^{3}\omega_j\frac{\partial^2\psi_j}{\partial z_j^2} = -\sum_{j=1}^{3}\omega_j\Delta\psi_j, \quad \tau_z = \Lambda\left(s_0 c_{44}\mathrm{i}\frac{\partial\psi_0}{\partial z_0} + \sum_{j=1}^{3}s_j\omega_j\frac{\partial\psi_j}{\partial z_j}\right), \tag{4b}$$

where the quantities U, σ_1, σ_2 and τ_z can be defined in the Cartesian coordinate (x,y,z) and the cylindrical coordinate (r,ϕ,z) in the complex forms as follows:

$$U = u + \mathrm{i}v = e^{\mathrm{i}\phi}(u_r + \mathrm{i}u_\phi), \quad \sigma_1 = \sigma_x + \sigma_y = \sigma_r + \sigma_\phi,$$
$$\sigma_2 = \sigma_x - \sigma_y + 2\mathrm{i}\tau_{xy} = e^{2\mathrm{i}\phi}(\sigma_r - \sigma_\phi + 2\mathrm{i}\tau_{r\phi}),$$
$$\tau_z = \tau_{xz} + \mathrm{i}\tau_{yz} = e^{\mathrm{i}\phi}(\tau_{zr} + \mathrm{i}\tau_{\phi z}), \tag{5}$$

where $z_j = s_j z \ (j = 0,1,2,3)$, $s_0 = \sqrt{c_{66}/c_{44}}$, $s_3 = \sqrt{\kappa_1/\kappa_3}$ and s_1 and s_2 are the two eigenvalues of following fourth degree equation:

$$a_0 s^4 - b_0 s^2 + c_0 = 0 \tag{6a}$$

where

$$a_0 = c_{33}c_{44}, \quad c_0 = c_{11}c_{44}, \quad b_0 = c_{11}c_{33} + c_{44}^2 - (c_{13} + c_{44})^2, \tag{6b}$$

$\psi_j \quad (j = 0,1,2,3)$ satisfy the following harmonic equations:

$$\left(\Delta + \frac{\partial^2}{\partial z_j^2}\right)\psi_j = 0 \quad (j = 0,1,2,3). \tag{6c}$$

where