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Analytical solutions of nonaqueous-phase-liquid dissolution problems associated with radial flow in fluid-saturated porous media

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SUMMARY

The removal of nonaqueous phase liquids (NAPLs) from contaminated soils by means of fresh water injection through a rejection well can be treated as a fully coupled problem involving the NAPL dissolution, radial aqueous-phase-liquid flow and dissolved NAPL transport through solute advection and diffusion/dispersion. The governing equations of this coupled problem can be mathematically described by a set of simultaneous partial differential equations with variable coefficients. In the case of the NAPL dissolution ratio (which is defined as the ratio of the equilibrium concentration of the dissolved NAPL to the density of the NAPL) approaching zero, analytical solutions for the NAPL dissolution problem associated with radial aqueous-phase-liquid flow have been derived in this paper. As a direct application example, the derived analytical solutions are used to investigate the fundamental behaviours of the NAPL dissolution problems associated with radial aqueous-phase-liquid flow in the fluid-saturated porous media. The related analytical results have demonstrated that three key factors, namely the dimensionless comprehensive number (which is known as the Zhao number and can be used to represent the overall hydrodynamic characteristic of a NAPL dissolution system), the initial saturation of the residual NAPL and the dimensionless injection well radius, can have significant effects on the dimensionless NAPL dissolution front propagation speed, the dimensionless NAPL dissolution front location and dimensionless breakthrough time of the NAPL dissolution front in the NAPL dissolution system associated with radial aqueous-phase-liquid flow.

1. Introduction

The removal of nonaqueous phase liquids (NAPLs) in contaminated subsurface soils has become an important topic in recent years (Miller et al., 1990, 1998; Geller and Hunt, 1993; Powers et al., 1994; Imhoff et al., 1994, 1996, 2002, 2003; Soerens et al., 1998; Willson et al., 1999; Seyedabbasi et al., 2008). When nonaqueous phase liquids (NAPLs), such as trichloroethylene, ethylene dibromide, benzene, and toluene (Miller et al., 1990), are released to groundwater, they can reside in the form of disconnected ganglia or blobs as residual saturations within the pores of porous media. Compared with the density of a NAPL such as trichloroethylene, the equilibrium concentration of the NAPL in the aqueous phase fluid is much smaller. For instance, the ratio of the equilibrium concentration of the dissolved NAPL to the density of the NAPL, which is called the NAPL dissolution ratio (Zhao et al., 2010), can be as small as 8.7×10^{-4} (Imhoff and Miller, 1996). This implies that the trapped NAPL can become a long-term source of groundwater contamination as a result of its low solubility. Due to this characteristic, analytical solutions were derived for NAPL dissolution problems associated with planar unidirectional flow in fluid-saturated porous media (Imhoff and Miller, 1996; Zhao et al., 2010). However, since the injection of fresh water through an injection well is one of the commonly-used techniques to remove the trapped NAPL from contaminated soils, it is desirable to derive analytical solutions for NAPL dissolution problems associated with radial flow in fluid-saturated porous media. Not only can the derived analytical solutions be used to understand the fundamental behaviours of NAPL dissolution problems associated with radial divergent flow, but also they can be served as benchmark solutions for verifying any computer codes that are used to solve NAPL dissolution problems associated with radial divergent flow in more complicated geometrical conditions. Due to their particular features of mathematical vigour and rigorous logic, analytical solutions are much more valuable than the corresponding numerical simulation solutions of approximate nature. This is the reason why analytical solutions are commonly used to check both the correctness and accuracy of the numerical simulation solutions, which are usually of approximate nature and can be obtained from the numerical simulation methods such as the finite element method and the finite difference method. For this reason, great

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efforts have always been made to derive analytical solutions for many scientific and engineering problems (Chen, 1985, 1987; Chen et al., 1996, 2002, 2007; Zhao and Steven, 1996; Zhao et al., 2010).

From the scientific point of view, NAPL dissolution problems associated with radial flow in fluid-saturated porous media can involve the following three main processes: the radial aqueous-phase-liquid flow process, the NAPL dissolution process, the dissolved NAPL transport process through both solute advection and diffusion/dispersion. This means that a NAPL dissolution source term needs to be included in the transport equation of the dissolved NAPL in the fluid-saturated porous medium. If both the NAPL dissolution process and the dissolved NAPL molecular diffusion process are neglected, the transport of the dissolved NAPL around a water injection well in the fluid-saturated porous medium can degenerate to the well-known radial dispersion problem, which is commonly defined as the problem of analyzing the dispersive transport of an aqueous contaminant or a tracer in steady-state radial flow from an injection well or a pumping well that fully penetrates a horizontal, homogeneous confined aquifer of uniform thickness and infinite lateral extent (Hoopes and Harleman, 1967; Sauty, 1980; Hsieh, 1986; Chen et al., 2002). For such a radial dispersion problem, an advection–dispersion equation involving the dependence of the hydrodynamic dispersion coefficient on the spatially varying velocity of the pore-fluid, which is, in essence, a partial differential equation with variable coefficients, is usually considered in a cylindrical coordinate system. Despite some mathematical difficulties, a considerable amount of research has been conducted to derive analytical solutions for the radial dispersion problems in fluid-saturated porous media (Tang and Basu, 1979; Sauty, 1980; Chen, 1985; Hsieh, 1986; Chen et al., 2002). The following two kinds of mathematical methods were used in the process of deriving the related analytical solutions: one is a combined method of the Laplace forward transform and inverse transform, while another is a combined method of the Laplace forward transform and power series expansion. In these two kinds of mathematical methods, the governing partial differential equation of the radial dispersion problem with variable coefficients can be first transformed to an ordinary differential equation with variable coefficients, which are in principle solvable using the conventional mathematical methods (Tang and Basu, 1979; Sauty, 1980; Hsieh, 1986; Chen et al., 2002). Although the Laplace forward transform solution can be straightforwardly obtained in the Laplace domain, it is often difficult to transform the Laplace forward transform solution into the Laplace inverse transform solution in the time domain. There are the following two ways to overcome this difficulty: one is to obtain the Laplace inverse transform solution using the numerical integration approach (Moench and Ogata, 1981; Chen, 1985, 1987; Chen and Woodside, 1988; Moench, 1989; Chen et al., 1996, 2007), while another is to directly solve the transformed ordinary differential equation with variable coefficients using the power series expansion approach (Chen et al., 2002, 2003; Chen, 2007), which is the standard approach for solving linear ordinary differential equations with variable coefficients. Nevertheless, to the best of the authors' knowledge, no analytical solution is available for NAPL dissolution problems associated with radial aqueous-phase-liquid flow in fluid-saturated porous media, so that it is desirable, in this paper, to derive some analytical solutions for this kind of NAPL dissolution problem.

Compared with the radial dispersion problem, the governing equations of the NAPL dissolution problem associated with radial aqueous-phase-liquid flow are much more complicated. For the radial dispersion problem, a single partial differential equation with variable coefficients is used to describe the governing equation of the problem, while for the NAPL dissolution problem associated with radial aqueous-phase-liquid flow, a set of simultaneous partial differential equations with variable coefficients needs to be used to describe the corresponding governing equations of the problem as a direct result of considering the NAPL dissolution process. Since the NAPL dissolution can cause a change in NAPL saturation, which in turn can lead to a variation in aqueous-phase-liquid relative permeability (Zhao et al., 2012), the radial aqueous-phase-liquid flow filed will be changed due to the NAPL dissolution in the NAPL dissolution system associated with radial aqueous-phase-liquid flow. On the other hand, a change in the radial aqueous-phase-liquid flow filed can affect the dissolved NAPL transport through advection and dispersion, so that the concentration distribution field of the NAPL species will be changed accordingly. Since the variations in both the radial aqueous-phase-liquid flow filed and the NAPL species concentration distribution field can influence the NAPL dissolution process (Zhao et al., 2010), a loop has been formed between the NAPL dissolution, radial aqueous-phase-liquid flow and dissolved NAPL transport through advection and diffusion/dispersion. This indicates that the NAPL dissolution problem with radial aqueous-phase-liquid flow is, in essence, a fully coupled problem involving the NAPL dissolution, radial aqueous-phase-liquid flow and dissolved NAPL transport through advection and diffusion/dispersion in the fluid-saturated porous medium. Due to the complicated and complex nature of this fully coupled problem, it is extremely difficult, if not impossible, to derive a complete set of analytical solutions for the general NAPL dissolution problem associated with the radial aqueous-phase-liquid flow. However, in some special case, such as in the case of NAPL dissolution ratio approaching zero, it is possible to derive analytical solutions for the NAPL dissolution problem associated with the radial aqueous-phase-liquid flow, as demonstrated in this study.

Keeping the above considerations in mind, the forthcoming contents of this paper are arranged as follows. In Section 2, the mathematical formulations of NAPL dissolution problems associated with radial aqueous-phase-liquid flow in fluid-saturated porous media are presented. In Section 3, analytical solutions to NAPL dissolution problems associated with radial aqueous-phase-liquid flow have been mathematically derived for a limit case, in which the NAPL dissolution ratio approaches zero. In Section 4, the derived analytical solutions are used to understand the fundamental behaviours of the NAPL dissolution problems associated with radial aqueous-phase-liquid flow in the fluid-saturated porous media. Finally, some conclusions are given in Section 5.

2. Mathematical model for NAPL dissolution problems associated with radial aqueous-phase-liquid flow

To describe NAPL dissolution problems associated with radial aqueous-phase-liquid flow, similar assumptions to those used in the NAPL dissolution problems, which are associated with planar aqueous-phase-liquid flow in two-dimensional fluid-saturated porous media (Imhoff and Miller, 1996; Zhao et al., 2010), are employed in this study. The related assumptions can be summarized as follows: (1) although NAPLs are mixtures of several species, only a single-species NAPL is considered; (2) water is negligible in the NAPL; (3) the NAPL is at residual saturation, existing as discrete ganglia or blobs trapped by capillary forces within the porous medium; (4) in the absence of the NAPL, the porous medium is homogeneous and isotropic; (5) the NAPL is immobile and incompressible; (6) water is incompressible and the solid matrix of the porous medium is rigid; (7) radial flow is in a horizontal plane of a homogeneous confined aquifer with a constant thickness so that gravity is neglected and a polar coordinate system is appropriate for describing the governing equations of the problem; (8) the aqueous phase moves through the region of the residual NAPL and slowly dissolves NAPL ganglia; and (9) the fresh water does not chemically react with other minerals to form precipitates that