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Experimental investigation on water flow in cubic arrays of spheres

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ABSTRACT

One-dimensional uniform flow in homogeneous porous media was experimentally investigated. Head drop experiments were conducted in four test tubes with cubic arrays of spheres in diameter 3 mm, 5 mm, 8 mm and 10 mm. The experimental results indicate that Darcy's law should be an approximate expression by neglecting the inertial term for flow at low velocity. Nonlinearity is attributed to inertial term in porous medium before the turbulent flow emerges. Forchheimer equation with constant coefficients can well predict the flow in porous medium. The relationship between the diameter of the particles and the coefficients a and b in the equations were verified. Different Ergun type equations were used to predict the head drop and compared to the experimental data. It shows that the Irmay equation could well predict the fluid flow in cubic arrays of spheres, while the prediction of head drop by Ergun equation was much higher than observed data. It indicates that the coefficients α and β in the Ergun type equations have certain relations with porosity or the pore structure and would vary for different medium. The discontinuity observed was interpreted by transition from steady flow to weakly turbulence and compared with previous studies.

1. Introduction

In 1855 Henry Darcy conducted a series of experiments on water flow through filter sands and established a linear equation:

$$u = -K\frac{\partial h}{\partial x} \quad (1)$$

in which u is the flow velocity, h the piezometric head, x denotes the length of the flow path, and K a factor of proportionality (Darcy, 1857). In a long time, flow in porous media was described by Darcy's law and standard software such as Modflow, Feflow was based on the linear law (Bear, 1988; Moutsopoulos et al., 2009).

For high velocity flow, obvious deviation from Darcy's law was firstly observed by Forchheimer and a non-linear equation with a quadratic term was presented (Forchheimer, 1901). The form of Forchheimer equation is:

$$-\frac{\partial h}{\partial x} = au + bu^2 \quad (2)$$

where a and b are constants of porous media and fluid. This quadratic law was confirmed by many experimental data (Dudgeon, 1966; Mathias, 2011; Qian et al., 2005).

Due to both the Darcy's law and the quadratic equation were empirical law from experimental data, theoretically derivation of the two equations starting from the Navier–Stokes equation was studied to give the empirical expressions more general physical formulation (Blick and Civan, 1988; Giorgi, 1997). Some investigators considered that non-linear behavior was attributed to turbulence (Bird et al., 1960; Ergun and Orning, 1949). But some investigators believed that non-linear behavior of flow at high velocity was attributed to the inertial forces which were caused by frequent changes in flow direction and the acceleration of fluids passing through the curved channels in porous media (Lindquist, 1933; Scheidegger, 1960). For low Reynolds number, the second-order term in Forchheimer equation can be neglected, and then the Forchheimer equation become Darcian. But at moderate Reynolds number, the inertial forces became significant gradually which cannot be neglected, and the Darcy' law is invalid (Moutsopoulos and Tsihrintzis, 2005; Wen et al., 2006). To better understanding the validity of the Darcy's law, the criterion for identifying non-linear flow was studied by various researches (Venkataraman and Rama Mohan Rao, 1998; Zeng and Grigg, 2006). However, the experimental data from previous studies (Ahmed and Sunada, 1969; Arbhabhirama and Dinoy, 1973) showed the relationship curve for Reynolds number versus friction factor is smooth with no sharp limit between laminar flow and transition flow.

In order to understand the effects of inertial on porous media flow, a number of numerical studies have been conducted in arrays of spheres. In the numerical investigation by Reynolds et al. (2000) through a face-centered cubic arrangement of spheres, critical Reynolds number for quasiperiodicity and chaos was discussed. Using theory and lattice-Boltzmann simulations, Hill et al.

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(2001a,b) examined the effects of fluid inertia on flows in simple cubic, face-centerd cubic and random arrays of spheres. The simulations show that first inertial contribution to the drag force decreases with increasing solid volume fraction (Hill et al., 2001a) and the simulations agreed with the Ergun's correlation at solid volume fractions approaching the close-packed limit (Hill et al., 2001b). Drag force under a variety of solid volume fractions and Reynolds numbers up to $Re = 1000$ were obtained by lattice-Boltzmann simulations for random arrays spheres by Beetstra et al. (2007). Applying the same technique, transitions from steady to unsteady chaotic flow in a close-packed face-centerd cubic array of spheres was examined by Hill and Koch (2002b). It indicated that the transition to unsteady flow occurred when the streamwise component of velocity fluctuates and reflectional symmetries were broken. To investigate 3D effect on flows through porous media, Fourar et al. (2004) calculated the viscous and the pressure drags on the spheres and evaluated their contributions to the deviation from Darcy's law by simulations of flow through arrays of spheres. van der Hoef et al. (2005) discussed the effects of diameter ratios, mass fractions and packing fractions on the drag force on the spheres by lattice-Boltzmann simulations of slow fluid flow past mono- and bidisperse random arrays of spheres. To know when to take inertia effects into account, Hellstrom et al. (2010) performed a CFD-based micromechanical investigation to investigate the limitations of the equations for flow through a quadratic array of cylinders packed.

The literature survey presented above shows there are many achievement in the simulations study on flow in array of spheres. However, there are no quantitative experimental studies examining the dynamics of such flows (Hill and Koch, 2002b). In this paper, we attempt to develop physical model with arrays of spheres to experimentally investigate the inertial effects on such flows and compare with the previous simulations. As report by Hill and Koch (2002a), there are some differences of hydrodynamic behavior between the random porous media and the ordered porous media. The fluid elements may undergo chaotic trajectories if the porous medium has a random structure, even in steady flows. In ordered porous media, long-range order can be broken by the onset of unsteady flow. Considering that it's difficult to make the theoretical and computational analysis of flows due to the complex pore structure of porous media, cubic arrays of spheres model was chosen to establish flow motion equation. Then, experiments on water flow in porous media made of spheres arrange in cubic were conducted to verify the theoretical equation, and determination of the coefficients of the equations were discussed. Inertial effects on flow obtained from experiments were compared with previous simulations. Finally, findings are summarized in the end.

2. Experimental set-up and preliminary test

2.1. Experimental apparatus

The experimental apparatus used in this investigation is shown in Fig. 1. It mainly consists of three parts: water supply device, measurement equipment and the test tubes. Water was provided by the centrifugal pump to the test tube and flow rate was adjusted by the inflow valve. Water passed through the test tube was collected by a cylindrical tank and the water-level fluctuation was measured to calculate the flow rate by pressure sensor (CY201) in range 0–20 kpa ±0.1% FS. Tap water was used as working fluid and its temperature was measured by thermometer ±0.1 °C. Each test tube is 3000 mm in length, and three pressure taps were set to measure the pressure drop. To avoid the inlet and outlet effects, two pressure taps were 250 mm away from both ends of the tube and a pressure taps in the middle. Pressure was measured by pressure sensor (CY201) in range 0–100 kpa ±0.1% FS. To avoid the effects of air on the experiments, follow measures were taken. Before starting experiments, we keep the test tube vertical and water was passed through the tube from the bottom to the top slowly to discharge air. The normal bend in the end of the tube (Fig. 1) keeped the tube full of water in the experimental process. Besides, the pressure taps could be used to discharge air if there was trapped air in the tube.

The test tubes were developed by organic glass with acrylic balls arranged in cubic (Fig. 2).

The acrylic balls having uniform size and shape were bonded by chloroform, which keep the structure of porous media from destroying under high water pressure. The fabrication of cubic arrays of spheres was simply introduced. Firstly, single balls were stuck into a square unit (Fig. 2a). Then square units were into cubic units which had stable structure (Fig. 2b). Finally, cubic units were filled in the Lucite tube (Fig. 2c). To keep the units immobile, cubic units on the both ends of the tubes were stuck to the wall of the tubes by chloroform. Flanged joint was used among the test tube, inflow tube and outflow tube (Fig. 2d). Four test tubes were made filled with balls in different size. The diameters of the balls were 3 mm, 5 mm, 8 mm and 10 mm. The porosity of the porous media with spheres arranged in cubic was equal to 0.4767, having nothing to do with diameter of the spheres.

2.2. Preliminary test

As the cubic arrays of spheres were filled in the tube, there will be friction between the fluids and the tube wall which is additional. The additional friction would influence the water head drop experimental results through porous media (Di Felice and Gibilaro, 2004; Eisfeld and Schnitzlein, 2001). So we conducted preliminary test to find out how to eliminate the wall effect before the official tests. Considering flow friction in porous media is relevant to the superficial area of particles (A_s), we can reduce the ratio between the area of tube wall (A_t) and the gross superficial area of spheres (A_s) in the cross section of the tube to decrease the wall effect. If we increase the number of balls in the cross section of test tubes, the wall effect may be neglected when the additional friction caused by the tube wall was much smaller than the flow friction in porous media. For cubic arrays of spheres, when the number of balls are $N \times N$ in the cross section of test tube, the area of tube wall (A_t) is $4(Nd^2)$ and the gross superficial area of balls (A_s) is $\pi d^2 N^3$. So the ratio between the area of tube wall (A_t) and the gross superficial area of balls (A_s) can be presented by formula:

$$\frac{A_t}{A_s} = \frac{4(Nd)^2}{\pi d^2 N^3} = \frac{4}{N\pi} \tag{3}$$

Eq. (3) shows that the value of A_t/A_s is inversely proportional to the number of balls in the tube wall (N), and has nothing to do with diameter of the balls. Based above analysis, we firstly designed and developed four test tubes with spheres 10 mm in diameter, and the number of balls in the cross section of the tubes were 2×2, 3×3, 4×4, 5×5 like that shown in Fig. 3.

Four groups of experiments were conducted in the tubes. The hydraulic gradient was calculated by formula:

$$J = \frac{\Delta h}{\Delta x} \tag{4}$$

in which Δx is length of the tube, Δh is friction head loss that can be measured experimentally. Velocity (u) can be determined by formula:

$$u = \frac{Q}{A} \tag{5}$$