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Reconstruction of soil thermal field from a single depth measurement

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SUMMARY

Soil field experiments usually consist of measurements of soil temperatures, heat fluxes and soil water contents. Accurate determination of the soil thermal field, in particular, prediction of the soil surface temperature and the ground heat, contains the signature to the surface energy partitioning, and is therefore critical to the surface energy balance closure problem. In this paper, we develop a numerical procedure to reconstruct the entire soil thermal field from a single depth measurement of either temperature or heat flux. The new algorithm is based on Green's function approach by using the fundamental solution of heat conduction in semi-infinite soils and Duhamel's integral for incorporation of general boundary conditions. It is highlighted that the new approach is capable of accurately reproducing results of some existing numerical approaches, with a more general setting and treatment of the heat diffusion problem, and hence provides a possible unified framework for the estimation of thermal field in soils.

1. Introduction

The transport of heat underneath the Earth's surface and the subsequent determination of the soil thermal field (viz. soil temperature and soil heat flux) are critical in regulating the subsurface and surface physical processes. In particular, as all major surface energy budgets (net radiative, sensible, latent and ground heat fluxes) are strong functions of the surface temperature, the subsurface heat transport largely dictates the partitioning of the available energy on the land surface (net radiation) into the dissipative heat budgets (sensible, latent and ground heat). A recent study by Bateni and Entekhabi (2012) showed that the land surface temperature can implicitly contain the signature to the surface energy partitioning through linear stability analysis. The accurate determination of the soil thermal field, therefore, is essential in establishing and assessing the surface energy balance closure and has attracted extensive research effort concerned with climate, weather and atmospheric dynamics (de Silans et al., 1997; Foken, 2008; Heusinkveld et al., 2004; McCumber and Pielke, 1981).

The subsurface thermal field can be constructed by solving the coupled heat, liquid water and vapor transport equations using advanced numerical techniques, such as the finite element method (FEM) (Bittelli et al., 2008; Vogel et al., 2011). On the other hand, numerous analytically-based approaches have been developed in past decades, to predict soil temperature and/or soil heat flux based on the one-dimensional (1D) heat diffusion, with applications to a wide range of areas including agronomy, meteorology, hydrology and ecology (Gao et al., 2003; Guaraglia et al., 2001; Holmes et al., 2008; Horton and Wierenga, 1983; Nunez et al., 2010). These models made use of analytical solutions of heat conduction in semi-infinite soils, and are numerically more economic and provide deeper insight into the subsurface physics as compared to FEM. In these models, certain time series of measured soil thermal properties, temperatures and heat fluxes at various depths are required as auxiliary data to complete estimations of the soil thermal quantities. Alternatively, the force-restore method, originally proposed for the derivation of prognostic surface temperature equations (Bhumralkar, 1975), together with its improved forms (Arya, 2001; Deardorff, 1978; Gao et al., 2008), were proven a powerful tool for soil thermal predictions. Instead of solving the second order partial differential equation, the heat diffusion process is simplified and represented as first order ordinary differential equations in the force-restore method, such that standard integration technique can be directly applied to obtain solutions of soil thermal field.

However, some theoretical aspects regarding the heat conduction process in soils remain obscure among researchers, leading to confusion in problem definition (Wang and Bou-Zeid, 2011), overdesign of auxiliary measurements (Wang and Bou-Zeid, 2012) and unnecessary restriction in model applicability (Wang and Bras, 1999). As a consequence, there are some major limitations inherent in most numerical models in the literature:

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(1) Force-restore methods, represented by first order differential equations, are inevitably simplifications of the exact heat diffusion process. At best, these solutions converge to the finite difference solution of the second-order diffusion equation (Deardorff, 1978).
(2) Boundary conditions of the 1D heat equation are usually prescribed using analytical functions, i.e. represented by sinusoidal forms or by Fourier series (Gao et al., 2003; Holmes et al., 2008; Horton and Wierenga, 1983; Nunez et al., 2010) whereas general boundary conditions representing more realistic natural forcing are not permissible.
(3) Auxiliary profile measurement of soil temperature, heat flux, and/or soil water content at different depths is required to construct the complete thermal field (Guaraglia et al., 2001; Kimball and Jackson, 1975; Liebethal et al., 2005), whereas only partial information of the thermal field can be reconstructed using measurements at a single depth (Wang and Bras, 1999).

In addition, while it is well-known that evolutions of the soil temperature and heat flux are two physically inseparable processes in the heat conduction (Carslaw and Jaeger, 1959), the possibility has long been overlooked that the complete thermal field information can be *encrypted* into a time series of a single thermal quantity (temperature or soil flux). This study is motivated from the prior work of the author (Wang and Bou-Zeid, 2012), in which the ground heat flux was estimated from the heat flux measurement at a single depth. The combination of Duhamel's principle and Green's function solutions of heat diffusion equation provide a powerful tool, rendering general solutions of 1D inhomogeneous heat diffusion problems analytically tractable. Here we further extend the Green's function approach in Wang and Bou-Zeid (2012) to reconstruct the soil thermal field from a single depth measurement of either temperature or heat flux (but not both).

The proposed method is validated by comparisons against the exact solution of a canonical 1D conduction problem. Furthermore numerical results are compared among predictions of the proposed and existing numerical methods, as well as field measurements of subsurface soil temperature and heat flux in a grassland area. The algorithm for reconstruction of the soil thermal field using a single depth measurement is shown to be robust and of good accuracy. We also show that the proposed method can reproduce estimations of soil temperature and soil heat flux by some commonly used numerical approaches with a more general problem setting. Therefore it provides a unique numerical algorithm, capable of embracing a family of existing numerical models in the literature under one unified framework. Moreover, the success of the new numerical algorithm sheds light on further development of this approach to solve, e.g. more general advection–diffusion process governing the coupled subsurface heat and soil moisture transport.

2. Mathematical model

In practice, the Earth's soil layer can be mathematically treated as a 1D semi-infinite solid domain with the spatial coordinate $0 \leqslant z < \infty$ (z is positive downwards). The 1D heat diffusion equation governing the evolution of the soil thermal field is given by

$$\frac{\partial T(z,t)}{\partial t} = \kappa \frac{\partial T^2(z,t)}{\partial z^2}, \tag{1}$$

where $\kappa = \lambda/\rho c$ is the soil thermal diffusivity, with λ, ρ and c the thermal conductivity, the density, and the specific heat of soil respectively. Note that in Eq. (1), we assumed the thermal diffusivity is constant across the depth of soil. While in general soil thermal properties are functions of soil moisture and soil temperature, the assumption of constant diffusivity is reasonable for most applications in soil physics, see, for example, the physical argument by Wang and Bras (1999) and the validation against experiments (Hanks, 1992; Wang and Bou-Zeid, 2012). It is also noteworthy that under extreme weather and climate conditions at the soil–atmospheric interface, such as during floods or in permafrost soils, in addition to heat diffusion, the contribution from moisture advection through porous soil layers will be significant (Gao, 2005; Heitman et al., 2010) and the complete heat transport process will be more realistically governed by the advection–diffusion equation. General formulation of the advection–diffusion problem for the coupled subsurface heat and moisture transfer is beyond the scope of this study, but remains an intriguing extension of the numerical algorithm proposed here.

For brevity, hereafter we denote any quantity involving both spatial and temporal variables as $\upsilon(z, t) = \upsilon_z(t)$, e.g. $G(z = 0, t) = G_0(t)$ represents the time series of the ground heat flux at $z = 0$. The boundary conditions (BCs) at the surface and the deep end of the soil are prescribed by heat flux forcing (Neumann boundaries), as

$$-\lambda \frac{\partial T_z(t)}{\partial z}\bigg|_{z=0} = f(t), -\lambda \frac{\partial T_z(t)}{\partial z}\bigg|_{z\to\infty} = 0, \tag{2}$$

respectively. Note that in Eq. (2), the boundary forcing $f(t)$ is an rather arbitrary function, provided it is square-integrable, i.e. well-defined in the L^2 (Hilbert) space. The initial condition, for simplicity, is prescribed by uniform temperature distribution inside the soil layer. This simplification has limited initial perturbation effect on the solution after a relatively short integration time (Wang and Bou-Zeid, 2012):

$$T_z(t = 0) = T_i. \tag{3}$$

In general, we can apply the Duhamel's principle (Carslaw and Jaeger, 1959) to solve the boundary value problem (BVP) presented in Eqs. (1)–(3). The temperature solution for the heat conduction in the semi-infinite domain is given by a Stieltjes integral, convoluting the boundary flux forcing and Green's function solution (Cole et al., 2011; Wang et al., 2011), as

$$T_z(t) = T_i + \int_0^t f(t-\tau) dg_z(\tau). \tag{4}$$

In Eq. (4), $g_z(t)$ is the Green's function solution of a homogeneous heat conduction problem corresponding to $f(t) = \delta(t)$, where $\delta(t)$ is the Dirac delta function, given by (Carslaw and Jaeger, 1959):

$$g_z(t) = \frac{2\sqrt{(\kappa t/\pi)}}{\lambda} \exp\left(-\frac{z^2}{4\kappa t}\right) - \frac{z}{\lambda} \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right), \tag{5}$$

where erfc(·) is the complimentary error function.

Combining Eqs. (4) and (5), and applying Fourier's law for heat conduction, the solution of soil heat flux can be calculated as

$$G_z(t) = -\lambda \frac{\partial T_z(t)}{\partial z}\bigg|_{z>0} = \int_0^t f(t-\tau) dF_z(\tau), \tag{6}$$

where

$$F_z(t) = \mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right), \tag{7}$$

is the fundamental solution of the soil heat flux of the 1D heat conduction with homogeneous BC. At the surface, $z = 0$, it is straightforward to show that Eq. (6) can be reduced to $G_0(t) \equiv f(t)$. This relation, as pointed out by Wang and Bou-Zeid (2011), physically preserves the energy conservation law in an infinitesimally thin layer of Earth's surface. Thus we have

$$T_z(t) - T_i = \int_0^t G_0(t-\tau) dg_z(\tau). \tag{8}$$