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Fractal scaling of apparent soil moisture estimated from vertical planes of Vertisol pit images

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SUMMARY

Image analysis could be a useful tool for investigating the spatial patterns of apparent soil moisture at multiple resolutions. The objectives of the present work were (i) to define apparent soil moisture patterns from vertical planes of Vertisol pit images and (ii) to describe the scaling of apparent soil moisture distribution using fractal parameters. Twelve soil pits (0.70 m long × 0.60 m width × 0.30 m depth) were excavated on a bare Mazic Pellic Vertisol. Six of them were excavated in April/2011 and six pits were established in May/2011 after 3 days of a moderate rainfall event. Digital photographs were taken from each Vertisol pit using a Kodak™ digital camera. The mean image size was 1600 × 945 pixels with one physical pixel ≈373 μm of the photographed soil pit. Each soil image was analyzed using two fractal scaling exponents, box counting (capacity) dimension (D_{BC}) and interface fractal dimension (D_i), and three prefractal scaling coefficients, the total number of boxes intercepting the foreground pattern at a unit scale (A), fractal lacunarity at the unit scale (Λ_1) and Shannon entropy at the unit scale (S_1). All the scaling parameters identified significant differences between both sets of spatial patterns. Fractal lacunarity was the best discriminator between apparent soil moisture patterns. Soil image interpretation with fractal exponents and prefractal coefficients can be incorporated within a site-specific agriculture toolbox. While fractal exponents convey information on space filling characteristics of the pattern, prefractal coefficients represent the investigated soil property as seen through a higher resolution microscope. In spite of some computational and practical limitations, image analysis of apparent soil moisture patterns could be used in connection with traditional soil moisture sampling, which always renders punctual estimates.

1. Introduction

Image analysis is a modern tool for quantifying the morphology of spatial patterns of soil properties. Many works have been conducted in this direction using both, 2D and 3D gray-level or binary (e.g. black and white) images. In particular, image analysis seems to be a useful tool for describing vegetative developmental stages (Behrens and Diepenbrock, 2006), texture recognition (Kilic and Abiyev, 2011) or quantification of differential growth processes in plant root and shoot growth zones (Chavarría-Krauser et al., 2007). Combinations of image and fractal analyses have been used for characterizing bulk density patterns (Zeng et al., 1996), soil macroporosity (Gantzer and Anderson, 2002), soil micromorphology (Bartoli et al., 2005) and soil structural state (Dathe and Thullner, 2005). In almost all previously cited studies, the box counting (capacity) dimension has been considered as the main parameter for characterizing the scaling behavior of the investigated soil property. Some recent works have also used multifractal measures for characterizing water fingering from magnetic resonance images (Posadas et al., 2009) and mass and entropy dimensions derived from 3-D soil images (Tarquis et al., 2008). In general, some other indices as fractal lacunarity and entropy scaling need to be incorporated as complementary parameters. Some studies have used fractal lacunarity for landscape texture evaluation (e.g. Plotnick et al., 1993) or scale-dependent clustering of fracture networks (Roy et al., 2010), but it is still a less considered scaling parameter. Even though most hydraulic soil properties are direct or indirect consequences of soil moisture distribution, there are relatively few works using 2-D or 3-D image analysis for describing soil moisture scaling. Due to the evident visual contrast among wetter and drier zones, image and fractal analyses can be useful for characterizing the local and global distribution of soil moisture within soil profiles. The objectives of the present work were (i) to define apparent soil moisture patterns from vertical planes of Vertisol pit images and (ii) to describe the scaling of apparent soil moisture distribution using fractal parameters.

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2. Theoretical considerations

Here we present a brief overview of the main fractal and pre-fractal parameters involved in the present work and the way they have been estimated.

2.1. Box counting (capacity) dimension

The use of box counting rationale for computing fractal (capacity) dimension assumes that the investigated pattern (e.g. soil moisture) fits some or all of the strong symmetries (e.g. translational, rotational and/or dilation invariance) (Feder, 1988) which can be called scaling invariance. For a 2-dimensional image one has to divide a Euclidean box of linear size, L, which contains the pattern, into $(L/r)^2$ smaller boxes each of linear size r. The number of non-overlapping boxes, N_i, of size $r_i \leqslant L$ containing pieces of the pattern (for example wet pixels) follows a power law:

$$N_i(L, r_i) \propto \left(\frac{L}{r_i}\right)^{D_{BC}} \tag{1}$$

where D_{BC} is the box counting or fractal capacity dimension. In practice, Eq. (1) represents a limit as $r_i \to 0$, which imposes some restrictions. For example, L represents the finite system size condition (upper cutoff) while always $r_i \neq 0$ (Baveye et al., 2008).

Eq. (1) was used with experimental data in the form:

$$N(r) = Ar^{-D_{BC}} \tag{2}$$

where A is a scaling coefficient accounting for the number of boxes intercepting the considered pattern (e.g. wet or dry zones) at the unit scale (e.g. $r \to 1$). One could note from Eq. (1) that:

$$A \propto L^{D_{BC}} \tag{3}$$

which connects the A coefficient directly with the fractal scaling of the investigated soil property or image pattern (D_{BC} in this case) and the initiator size, L. A log–log transformation of Eq. (2) allows one to estimate D_{BC} as the slope of the linear regression equation and log(A) as the corresponding intercept. That is:

$$\log(N) = \log(A) - D_{BC}\log(r) \tag{4}$$

To our knowledge, only Kravchenko et al. (2011) and few other workers have paid attention to the potential utility of the A coefficient as another scaling constant. Thus, within the context of the present work the A coefficient is used for estimating a corrected value of the apparent soil moisture.

2.2. Interface fractal dimension

Interfaces are geometrical structures separating two or more phases in soil system (e.g. pore–solid or dry–wet interface). From a theoretical point of view, a real world interface mimics, to some extent, the random counterpart of the deterministic von Koch curve. Many important physical, chemical and biological phenomena occur just at those boundaries. In principle, the box counting fractal theory is appropriate for estimating the complex geometry of such boundaries. Thus, Eqs. (1), (2), and (4) are valid for a quantitative description of such irregular interfaces. However, in this case $D_{BC} = D_i$ (the interface fractal dimension) and $N = N_s(r)$ is the number of boxes covering the interface at each resolution, r. For the case of the dry–wet frontier, the $N_s(r)$ value can be calculated using the same equation in Dathe and Thullner (2005):

$$N_s(r) = N_w(r) + N_d(r) - N_{\max}(r) \tag{5}$$

where N_w and N_d are the number of boxes covering wet and dry zones, respectively, and $N_{\max}$ is the total number of boxes covering the entire image at each resolution, r. In particular, $N_{\max}(r)$ can be calculated using a simple equation:

$$N_{\max}(r) = \frac{N_p}{r_i^2},\ \ \mathrm{r_i} = 1, 2, 4, \ldots, \mathrm{L} \tag{6}$$

where N_p is the total number of pixels covering the image.

2.3. Fractal lacunarity

Fractal lacunarity is a complementary measure for objects with similar fractal dimensions (Mandelbrot, 1983; Kaye, 1989). Allain and Cloitre (1991) defined lacunarity as a scale-dependent measure of heterogeneity of an object, whether or not it is fractal. In other words, it is the deviation of a fractal object from translational invariance. Briefly, lacunarity conveys information on the density of occupation of massless zones within the fractal object. For the sake of completeness, the term succolation is also complementary to lacunarity for fractal systems where percolation can occur (e.g. soils). According to Mandelbrot (1977) definition, a succolating system is one close to percolation. The theoretical background for computing lacunarity using the gliding box method is reported in many papers (e.g. Allain and Cloitre, 1991; Plotnick et al., 1993; Baveye et al., 2008; Przemyslaw, 2009). We reproduce it briefly within the context of binary images.

A box of size r is positioned at the origin of the binary image. As the box moves (e.g. moving window) through the image, it is calculated the number, n, of black pixels within the box at each position (let us assign a "mass", m, to this box). This procedure renders a frequency distribution function $n(m,r)$ which is converted into a probability distribution function $P(m,r)$ after dividing by the total number of boxes $N(r)$ of size r. Now, the first (Q_1) and second (Q_2) order statistics of the distribution can be determined as:

$$Q(1) = \sum mP(m, r) \tag{7}$$

$$Q(2) = \sum m^2 P(m, r) \tag{8}$$

The lacunarity for the specific box size was computed as:

$$\Lambda(r) = 1 + \frac{\sum m^2 P(m, r)}{\left[\sum mP(m, r)\right]^2} \tag{9}$$

Note that for a non-lacunar structure (e.g. translationally invariant) $\Lambda(r) = 1$, which suggests that lacunarity is statistically a measure of the distribution width (Baveye et al., 2008).

An interesting question refers to the scaling of $\Lambda(r)$ as a Pareto law of r.

$$\Lambda(r) = \Lambda_1 r^{-b} \tag{10}$$

Here we interpret Λ_1 as the lacunarity at a unit scale while b is a scaling exponent. Both, Λ_1 and b can be estimated from the log–log transformation:

$$\log \Lambda(r) = \log \Lambda_1 - b \log r \tag{11}$$

2.4. Scale-dependent Shannon entropy

The distribution of white/black pixels within a 2-dimensional soil image can be very heterogeneous. In fact, Shannon entropy might be an interesting informational measure of the effective measure of the investigated distribution. Here, one takes the advantage that a soil image can be partitioned into several boxes of sizes $r_1, r_2, \ldots, r_n$.

For discrete distributions, the representation of the Shannon entropy as a function of the box size, r, is:

$$S(r) = -\sum_{i=1}^{n} p_i(r) \log p_i(r) \tag{12}$$