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Open problems in finite projective spaces

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ABSTRACT

Apart from being an interesting and exciting area in combinatorics with beautiful results, finite projective spaces or Galois geometries have many applications to coding theory, algebraic geometry, design theory, graph theory, cryptology and group theory. As an example, the theory of linear maximum distance separable codes (MDS codes) is equivalent to the theory of arcs in $\mathrm{PG}(n,q)$; so all results of Section 4 can be expressed in terms of linear MDS codes. Finite projective geometry is essential for finite algebraic geometry, and finite algebraic curves are used to construct interesting classes of codes, the Goppa codes, now also known as algebraic geometry codes. Many interesting designs and graphs are constructed from finite Hermitian varieties, finite quadrics, finite Grassmannians and finite normal rational curves. Further, most of the objects studied in this paper have an interesting group; the classical groups and other finite simple groups appear in this way.
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1. Introduction

Apart from being an interesting and exciting area in combinatorics with beautiful results, finite projective spaces or Galois geometries have many applications to coding

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theory, algebraic geometry, design theory, graph theory, cryptology and group theory. As an example, the theory of linear maximum distance separable codes (MDS codes) is equivalent to the theory of arcs in $\mathrm{PG}(n,q)$; so all results of Section 4 can be expressed in terms of linear MDS codes.

Finite projective geometry is essential for finite algebraic geometry, and finite algebraic curves are used to construct interesting classes of codes, the Goppa codes, now also known as algebraic-geometry codes. Many interesting designs and graphs are constructed from finite Hermitian varieties, finite quadrics, finite Grassmannians and finite normal rational curves. Further, most of the objects studied in this paper have an interesting group; the classical groups and other finite simple groups appear in this way.

For more history, see the next section, taken from the preface of [67]. For a collection of current topics of interest, see [39].

2. Notation

$\mathbf{F}_q$	the finite field of order q
$\mathrm{PG}(n,q)$	the projective space of n dimensions over $\mathbf{F}_q$
$V(k,q)$	the vector space of k dimensions over $\mathbf{F}_q$

3. History

Associated to any topic in mathematics is its history. The first actual reference or near-reference on finite geometry is von Staudt's *Beiträge* [153] (1856). It contains a discussion of the notion of *real* and *complex* points of a finite space, which are counted as if they were points over $\mathbf{F}_q$ and $\mathbf{F}_{q^2}$; only dimensions two and three are considered. Then Fano [51] (1892) defined $\mathrm{PG}(n,p)$ synthetically, while more than a decade later Hessenberg [61] (1903) did it analytically. Next, Veblen and Bussey [155] (1906) gave the first systematic account of $\mathrm{PG}(n,q)$ for any n and q. However, it may be noted that the group $\mathrm{PGL}(n+1,q)$ of projectivities, which is implicit in the geometry, goes back to Jordan [83] (1870).

At the same time and later, Dickson [47,48] was investigating modular invariants, curves and other parts of algebraic geometry over a finite field. The link with statistics was developed by Bose [23] (1947); earlier, Fisher [54] (1942) had produced an experimental design from a finite plane, with Yates [159] (1935) already having made the connection with block designs. An essential problem is the determination of the numbers $m(N;r,s;n,q)$, which are related to these theories; they are defined below. To approximate real Euclidean space, Kustaanheimo [87] (1950) used finite spaces with the specific aim of verifying some fundamental constants; so, Finnish astronomers were thus led to explore finite spaces. Several of these strands were brought together when Segre, rising to Kustaanheimo and Järnefelt's challenge [82], showed [111] in 1954 his result that yielded many more on the same theme, but still leaves many easily stated but seductively difficult problems unsolved. Arising out of Segre's theorem, a typical problem is to deter-

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