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On the support algebras of indecomposable modules over tame algebras of polynomial growth ☆

Adam Hajduk

Faculty of Mathematics and Computer Science, Nicolaus Copernicus University, Chopina 12/18, 87-100 Toruń, Poland

A R T I C L E I N F O

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A B S T R A C T

We investigate the support algebras of finite dimensional indecomposable modules over representation-infinite tame algebras of polynomial growth over an algebraically closed field.

0. Introduction and the main result

Throughout this paper, by an algebra we mean a basic, finite dimensional k-algebra (associative, with identity) over an algebraically closed field k. For an algebra A, we denote by $\mathrm{mod}\, A$ the category of finite dimensional (over k) right A-modules, and by

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E-mail address: ahajduk@mat.umk.pl.

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ind A the full subcategory of mod A formed by the indecomposable modules. It follows from general theory that every algebra A is isomorphic to a bound quiver algebra kQ/I, where $Q = Q_A$ is the Gabriel quiver of A, and I is an admissible ideal in the path algebra kQ of Q over k (see [1, Chapter II]).

From the remarkable Tame and Wild Theorem of Drozd [18] (see also [11]), the class of finite dimensional algebras over an algebraically closed field k may be divided into two disjoint classes. The first class is formed by the *tame algebras* for which the indecomposable modules occur in each dimension in a finite number of discrete, and a finite number of one-parameter families. The second class is formed by the *wild algebras*, whose representation theory comprises the representation theories of all finite dimensional algebras over k (see [24, Chapter XIV] for a discussion). Accordingly, we may realistically hope to classify the indecomposable finite dimensional modules only for the tame algebras. A distinguished class of tame algebras is formed by the *representation-finite algebras*, having only a finite number of isomorphism classes of indecomposable modules. The representation theory for this is already well understood (see [3,4,7,9]). On the other hand, the representation theory of arbitrary tame algebras is still only emerging. Presently, the most accessible seems to be the class of tame algebras of *polynomial growth* [25], for which there exists a positive integer m such that the indecomposable modules occur in each dimension d in a finite number of discrete, and at most d^m one-parameter families.

Frequently, by applying covering techniques and geometric deformations [17,19–21, 27], we may reduce the representation theory of a given tame algebra to that for the corresponding tame simply connected algebras. This is the case for all representation-finite algebras (see [4,10]) and the self-injective algebras of polynomial growth (see [26]). Recall that, following [2], an algebra A is called *simply connected* if it is triangular (the Gabriel quiver Q_A of A is acyclic) and, for any presentation $A \cong kQ/I$ of A as a bound quiver algebra, the fundamental group $\Pi_1(Q, I)$ of (Q, I) is trivial. Then, it is natural to split the algebras into two classes: the *standard algebras*, which admit simply connected Galois coverings, and the remaining *nonstandard algebras*. The nonstandard representation-finite algebras occur only for algebraically closed fields k of characteristic 2, and are described completely (see Section 1 for more details). On the other hand, there are representation-infinite nonstandard algebras of polynomial growth for algebraically closed fields k of any characteristic (see [6]).

Recently, Skowroński raised the problem of describing the isomorphism classes of all nonstandard representation-infinite tame algebras of polynomial growth. The main result of this paper is a step towards the solution for this problem.

Let A be an algebra and M a module in ind A. Important information concerning the structure of M is coded into the structure and properties of its *support algebra* $\mathrm{supp}(M) = A/A(1-e)A$, where e is an idempotent of A such that the simple summands of the semisimple module $eA/e\,\mathrm{rad}\,A$ are exactly the simple composition factors of M. Clearly, M is a right module over $\mathrm{supp}(M)$. If $\mathrm{supp}(M) = A$, then the module M is said to be a *sincere module*. Finally, an algebra A is said to be *sincere algebra* if there is a sincere indecomposable A-module.