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# $*$-Superalgebras and exponential growth

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ABSTRACT

In this paper, we study the exponential growth of $*$-graded identities of a finite dimensional $*$-superalgebra $A$ over a field $F$ of characteristic zero. If a $*$-superalgebra $A$ satisfies a non-trivial identity, then the sequence $\{c_n^{\mathrm{gri}}(A)\}_{n\geq 1}$ of $*$-graded codimensions of $A$ is exponentially bounded and so we study the $*$-graded exponent $\exp^{\mathrm{gri}}(A) := \lim\limits_{n\to\infty} \sqrt[n]{c_n^{\mathrm{gri}}(A)}$ of $A$. We show that $\exp^{\mathrm{gri}}(A) = \dim_F(A)$ if and only if $A$ is a simple $*$-superalgebra and $F$ is the symmetric even center of $A$. Also, we characterize the finite dimensional $*$-superalgebras such that $\exp^{\mathrm{gri}}(A) \leq 1$ by the exclusion of four $*$-superalgebras from $\mathrm{var}^{\mathrm{gri}}(A)$ and construct eleven $*$-superalgebras $E_i, i = 1, \ldots, 11$, with the following property: $\exp^{\mathrm{gri}}(A) > 2$ if and only if $E_i \in \mathrm{var}^{\mathrm{gri}}(A)$, for some $i \in \{1, \ldots, 11\}$. As a consequence, we characterize the finite dimensional $*$-superalgebras $A$ such that $\exp^{\mathrm{gri}}(A) = 2$.



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## 1. Introduction

Let $A$ be an associative algebra over a field $F$ of characteristic zero. It is well known that if $A$ is a PI-algebra, i.e. if $A$ satisfies a non-trivial ordinary identity, then the sequence $\{c_n(A)\}_{n\geq 1}$ of codimensions of $A$ is exponentially bounded, i.e. there exist constants $a, k$ such that $c_n(A) \leq ak^n$, for all $n \geq 1$ [10]. In the 1980's, Amitsur conjectured that the limit $\exp(A) := \lim_{n\to\infty} \sqrt[n]{c_n(A)}$, called exponent of $A$, exists and is a non-negative integer. In 1999, Giambruno and Zaicev [4,5] confirmed this conjecture and provided an explicit formula to compute the exponent of any PI-algebra $A$.

It is natural to ask if a similar phenomenon is true in the setting of algebras with some additional structure, e.g. algebras with involution and $G$-graded algebras, where $G$ is a finite group. In [6] Giambruno and Zaicev confirmed the analog of Amitsur's conjecture for finite dimensional algebras with involution and in [1] Aljadeff and Giambruno confirmed the analog of Amitsur's conjecture for $G$-graded PI-algebras, where $G$ is a finite group.

Later, in [8] Gordienko considered the so-called algebras with a generalized $H$-action. Algebras with involution and $G$-graded algebras are particular cases of algebras with a generalized $H$-action. In this paper he confirmed the analog of Amitsur's conjecture in case $A$ is a finite dimensional algebra with a generalized $H$-action.

In [3], Giambruno, dos Santos and Vieira studied a new class of algebras: $*$-superalgebras. A $*$-superalgebra is a superalgebra $A = A^{(0)} \oplus A^{(1)}$ endowed with an involution which preserves the homogeneous components $A^{(0)}$ and $A^{(1)}$. By following the analogous procedure of the ordinary case, they studied the behavior of $*$-graded codimensions $c_n^{\mathrm{gri}}(A)$ of a $*$-superalgebra $A$ and proved that if $A$ is a PI-algebra, then $c_n^{\mathrm{gri}}(A)$ is exponentially bounded. Also, they classified the finite dimensional $*$-superalgebras of polynomial growth, i.e. $*$-superalgebras $A$ such that $c_n^{\mathrm{gri}}(A) \leq an^k$, for some constants $a, k$, for all $n \geq 1$, by excluding four $*$-superalgebras from $\mathrm{var}^{\mathrm{gri}}(A)$, the $*$-supervariety generated by $A$. Since a $*$-superalgebra $A$ can be viewed as an algebra with a generalized $FG$-action, where $G = \mathbb{Z}_2 \times \mathbb{Z}_2$ acts on $A$ by automorphisms and antiautomorphisms, the analog of Amitsur's conjecture was confirmed for $*$-superalgebras in [8].

This paper is devoted to the study of the exponential growth of $*$-graded codimensions and the $*$-graded exponent $\exp^{\mathrm{gri}}(A) := \lim_{n\to\infty} \sqrt[n]{c_n^{\mathrm{gri}}(A)}$ of a finite dimensional $*$-superalgebra $A$ over a field $F$ of characteristic zero. First, we characterize the finite dimensional simple $*$-superalgebras as those $*$-superalgebras such that $\exp^{\mathrm{gri}}(A) = \dim_F(A)$ and the field $F$ coincides with the symmetric even center of $A$. After, as a consequence of the results proved in [3], we characterize the finite dimensional $*$-superalgebras such that $\exp^{\mathrm{gri}}(A) \leq 1$. In the last section, we give a characterization of finite dimensional $*$-superalgebras such that $\exp^{\mathrm{gri}}(A) > 2$ by excluding a finite number of $*$-superalgebras from $\mathrm{var}^{\mathrm{gri}}(A)$. As a consequence, we characterize the finite dimensional $*$-superalgebras such that $\exp^{\mathrm{gri}}(A) = 2$.